

1 **Natural mortality estimations for beaked redfish (*Sebastes***
2 ***mentella*) - a long-lived ovoviviparous species of the**
3 **Northeast Arctic**

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9 **Abstract**

10 Beaked redfish (*Sebastes mentella*) is one of the few commercially important fish species
11 producing living offspring. Like many other deep-water fishes, it is slow growing, late maturing
12 and long-lived, getting as old as 65 years or even 75 years in some populations. The stock in the
13 Norwegian and Barents Seas is analytically assessed since 2012, assuming a natural mortality
14 (M) of 0.05 yr^{-1} across all ages, based on Hoenig's longevity related estimator. Since then,
15 several reviews and new studies have been published, affording a re-evaluation of the natural
16 mortality in the assessment model. We estimated natural mortality of beaked redfish using 48
17 different estimators in two categories, either estimating M across the population or dependent on
18 age, length or weight. The obtained estimates were highly variable, ranging from 0.01 to 0.32 yr^{-1}
19 ¹ for estimates across the population, and 0.05 to 0.62 yr^{-1} for age or size dependent median- M .
20 These wide ranges demonstrate the sensitivity of SSB to the input parameters as well as the
21 pitfalls of using estimators developed for distant taxa. Of the 48 estimates, 20 were considered as
22 potential candidates for updating the stock assessment model, yet not all of these were realistic.

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23 The frequency distribution of the estimators showed a peak around a value of 0.07 yr^{-1} , lowering
24 the estimated spawning stock biomass in the last year by about 300 kt but keeping it above the
25 precautionary reference point for the stock. Estimators across the life-history of beaked redfish
26 indicate a potential underestimation of natural mortality for early and overestimation for late age
27 classes. Age and size related estimators are promising for the older ages but tend to extreme
28 estimates for the early ages, which deserves further investigation.

29 **Keywords:** Single stock assessment, Norwegian Sea, Barents Sea, life history correlates,
30 ecological theory

31

32 1 Introduction

33 Fish stock assessment models are tools designed to describe the dynamics of exploited
34 populations that suffer mortality from natural causes (M) and from fishing (F). The two sources
35 of mortality sum up to total mortality (Z) with the simple equation $Z = M + F$. Consequently, an
36 accurate estimate of M is required to correctly estimate F and by extension Z , to give reliable
37 advice to fisheries. The current stock assessment for beaked redfish (*Sebastes mentella*) in the
38 Norwegian and Barents Seas uses a statistical catch-at-age (SCA) model which requires natural
39 mortality-at-age as input information. In the current stock assessment M is assumed to be 0.05
40 yr^{-1} across all years and ages (WKREDFISH, ICES 2018a). The value is derived from a
41 longevity of 75 years related to M following Hoenig's approach (Hoenig 1983). Given the
42 longevity of beaked redfish (Campana et al. 1990), it is desirable to implement an estimate of M
43 that accounts for its decline that can be observed for older and bigger fish (Caddy 1991). Stock
44 assessment models for species managed within the framework of ICES (International Council for
45 the Exploration of the Sea) are revised in a benchmark workshop that should achieve a consensus
46 for the way a stock is assessed in the next several years. For beaked redfish in ICES areas 1 and
47 2, corresponding to the Norwegian and Barents Seas, the last benchmark workshop was in 2018
48 (ICES 2018a) with the next revision planned for 2024.

49 Estimating M for wild populations is a challenging task and it is often boiling down to widely
50 accepted values such as 0.2 (Brodziak et al. 2011; Pope et al. 2021; Cope and Hamel 2022).
51 Getting more accurate estimates of M can take several approaches, the majority of which
52 demands a lot of data and extensive prior knowledge of the considered stock's biology, neither of
53 which is available for many fisheries, particularly along the coasts of the global South
54 (Kenchington 2014). A statistical approach is to estimate Z from catch curves for several periods
55 with different, but within the period stable, F and then extrapolate to zero fisheries mortality
56 (Ricker 1975). This can also be done in the absence of extensive age data by using length-based
57 estimates of Z as suggested by Beverton (1956). Direct measurements of M can be achieved with
58 tag-recapture experiments (Quinn and Deriso 1999), but for reliable estimates the cost of the
59 required tagging program can be prohibitive (Kenchington 2014). A rare case, because it requires
60 precise catch and survey data, is to estimate fisheries as the ratio of catch to fishable biomass and
61 then derive M from the estimates of F and Z (Xucai et al. 1996). Also requiring a data-rich

62 background are estimates of M from multispecies virtual population analysis (VPA) or in single-
63 species assessment models (Magnusson 1995; Maunder and Wong 2011).

64 However, for the majority of fish stocks there is not enough data to use these more sophisticated
65 methods and ever since Beverton (1963) there are attempts to estimate M based on theoretical or
66 empirical correlates between natural mortality and life history traits such as life expectancy, age
67 at maturity or animal growth rates. These efforts have led to a diverse range of methods which
68 Cubillos, Alarcón, and Brante (1999) summarized under the term ‘empirical methods,’ although
69 Kenchington (2014) later noted that these methods also include some that are solely based in
70 theoretical considerations.

71 Kenchington’s recent review (2014) provides a broad overview of M estimators based on age,
72 life history correlates and ecological theory. Additional estimators, or revisions of earlier ones,
73 have also been proposed by Hamel (2015), Hamel and Cope (2022), Then et al. (2015) and
74 Mangel (2017). Torrejón-Magallanes et al. (2021) introduced a development of Caddy’s
75 gnomonic approach (1991, 1996) and *gnomonicM* an associated R-package (Torrejón-
76 Magallanes 2021). This method divides the lifespan of a species into intervals that become the
77 longer, the more time has elapsed since birth, maintaining a constant proportion between an
78 interval’s duration and the elapsed time.

79 The objective of this short communication is to explore a range of natural mortality estimators
80 for the population of *S. mentella* in the Norwegian and Barents Seas and to provide revised
81 estimates of M for the different age groups that compose the population, which can serve as
82 input for the assessment of this fish stock. For this purpose, we compile a number of estimators
83 of M , either for the whole population or by age, length or weight. We use data collected by the
84 Institute of Marine Research in Norway as the source of life history parameters that can be used
85 to derive mortality estimates.

86 **2 Materials and Methods**

87 **2.1 Life history parameters**

88 Many empirical estimators of M are derived from other life history parameters. Some of these
89 parameters refer to the adult component of the population while others are given for adults and
90 juveniles. We extracted growth, fecundity and other life history parameters for the relevant
91 population components from a number of sources (Aanes 2012; Planque and Nedreaas 2010;
92 Nedreaas 1990; St-Pierre and De Lafontaine 1995; Saborido-Rey et al. 2015). Which life history
93 parameters are used, depends on the assumptions made for the considered estimators (Figure 1).
94 The parameters set up for estimators that assume $t_0 = 0$ are following Aanes (2012), estimators
95 covering adult and juvenile fish use the parameters by Nedreaas (1990) and those that cover the
96 adult population are parameterized following the report from the DeepFishMan project (Planque
97 and Nedreaas 2010). In that latter case, an age range that covers the bulk of the adult population
98 (age-11 to age-40) was used.

99 **2.2 Natural mortality estimators**

100 Following Kenchington (2014) natural mortality estimators can be categorized into i) age-based
101 estimators, most of them for M (e.g. Sekharan 1974; Charnov and Berrigan 1990) but some also
102 for Z such as Hoenig's estimator (Hoenig 1983), ii) life history based ones (e.g. Pauly 1980;
103 Gislason et al. 2010; Mangel 2017) and iii) those based on ecological theory (e.g. Jensen 1996;
104 Charnov et al. 2013). This is only one possible categorization, as several estimators contain age
105 as well as growth parameters (e.g. Alverson and Carney 1975; Chen and Watanabe 1989) and
106 those derived from ecological theory are often similar to those based on life history or regression
107 (e.g. Then et al. 2015; Roff 1984). Another common categorization is following input
108 parameters, which in case of this study would have resulted in a confusing number of categories.
109 Many estimators are further developments or updates of earlier ones and could be interpreted as
110 families of estimators. In total we applied 48 estimators (45 estimators + 3 variants of the
111 gnomonic approach), drawn from Kenchington's review (2014), Hamel (2015), Hamel and Cope
112 (2022), Then et al. (2015), and Mangel (2017). Three of these estimators (Hoenig 1983; Bayliff
113 1967; Kenchington 2014) estimate Z and the rest provides estimates of natural mortality (M). Of
114 the estimators 17 each fall into the age-based and life history based groups and 14 are based on

115 ecological theory. The estimators use a large number of parameters which are described in Table
116 1, whilst detailed descriptions of each estimator and their equations, and an explanation of why
117 they were excluded or retained, can be found in the supplementary material.

118 **2.3 Comparison of estimates**

119 Estimates of natural mortality rates were qualitatively assessed for their applicability to beaked
120 redfish. The criteria were, testability of their assumptions, suitability of the underlying data sets
121 and whether they were superseded by a later re-evaluation. In relation to the assessment of the
122 beaked redfish in ICES areas 1 and 2, i.e. the Norwegian and Barents Seas (ICES 2021) we
123 compared the estimates with the currently used value of $M = 0.05 \text{ yr}^{-1}$ and also tested the
124 sensitivity of the assessment model to the different estimates, using either point values or vectors
125 of values across ages, for those estimators that vary with age or size. Evaluations during the
126 benchmark workshop showed that the log-likelihood estimations of the model parameters are
127 robust against natural mortality values of $0.00 < M < 0.75$, whilst spawning stock biomass was
128 sensitive to different rates of M (WKREDFISH, ICES 2018a). Therefore, only SSB was used to
129 test how the mortality estimators considered as applicable affected the assessment. The model
130 settings, input data and assessment period were used as in the benchmark assessment except for
131 the use of fixed weights-at-age across years, which was a modification added at the arctic
132 fisheries working group (AFWG) in the same year (ICES 2018a, 2018b). A modification made
133 for this study was to recode the model to accept a vector of M -values across ages. The recoded
134 model was tested with a vector of 18×0.05 for ages 2-19+, getting the same results as with the
135 unmodified model using a single value. A brief description of the assessment model is given in
136 the supplemental material, whilst a detailed description can be found in the annex of the
137 benchmark workshop report (ICES 2018a).

138 **3 Results**

139 **3.1 Natural mortality estimates**

140 The estimates obtained with the 48 candidate estimators ranged from a very low value of 0.014
141 yr^{-1} for Alverson and Carney's estimator to a value of 0.617 yr^{-1} for Jennings and Dulvy's

142 estimator (Table 2). Although the latter value is a median value across the range of considered
143 weights (378 g - 1063 g, corresponding to the ages 11 (t_m) to 75 (T_{max}) it does not get lower
144 than 0.58 even for the largest fish.

145 **3.1.1 Age-based estimators**

146 Of the 17 age-based estimators 7 were considered applicable to beaked redfish, ranging from
147 0.019 yr⁻¹ for Zhang and Megrey's estimator to 0.2 yr⁻¹ for Charnov and Berrigan's estimator.
148 Alverson and Carney's estimator was even lower, but as Zhang and Megrey's estimator is a
149 revision of this estimator, we considered it preferable. Several of the eight 'not applicable'
150 estimators were excluded because they were established using data from small, short-lived,
151 pelagic fish, this applies to Bayliff's, Sekharan's and Tanaka's estimators. The other ones were
152 excluded because their assumptions could not be tested (Kenchington's estimator), information
153 on required parameters was too uncertain (Chen and Watanabe's estimator and Roff's first
154 estimator) or because the authors themselves did not recommend the use of the estimator in
155 assessment (Jensen's first estimator). Then's first estimator was regarded as not applicable as it
156 is superseded by an updated version of Hamel's first estimator, and Then's third estimator should
157 be used with caution as it is biased towards small, short-lived fish with high mortality rates.

158 **3.1.2 Life history correlates**

159 Life history correlates gave estimates for mortality rates between 0.034 yr⁻¹ and 0.322 yr⁻¹,
160 corresponding to Alagarajas and Jensen's third estimator, respectively. Of these estimators six
161 were excluded, all of them because they were based on very different taxa or in case of Djabali's
162 estimator on a different climate zone, i.e. the Mediterranean zone. Cubillo's estimator was not
163 completely excluded but must be used cautiously since $T_{95\%}$ is difficult to define. Likewise,
164 Then's fifth estimator should be used cautiously as it is biased by greater weights assigned to
165 short-lived species. Notable is the prevalence of the von Bertalanffy coefficient K in this group
166 that occurs in all but those estimates that solely use the gonadosomatic index (GSI).

167 **3.1.3 Estimates based on ecological theory**

168 The estimates based on ecological theory were the overall highest estimates, with 9 out of 11
169 crossing the 0.1 yr⁻¹ threshold. However, this may be skewed by the high number of weight-

170 dependent estimators in this group which give extremely high estimates for early ages with low
171 body-mass. A notable exception from this pattern was Charnov's estimator which yielded
172 mortality rates between 0.070 yr^{-1} and 0.046 yr^{-1} for the bulk of the adult population between 11
173 and 40 years of age. Only three of the estimators were excluded. Firstly, Then's fourth estimator
174 because it is superseded by an update of Hamel's second estimator. Secondly, Peterson &
175 Wroblewski's estimator because the size spectra it considers includes everything from
176 zooplankton to whales. Finally, Groeneveld's estimator because of high uncertainty in the
177 estimate. However, the proportion of estimators that need to be used with caution, seven out of
178 eleven, is higher than for the other groups.

179 **3.2 Comparison of estimates**

180 The frequency distribution of the point estimate mortality rates shows a peak at 0.065 yr^{-1} for all
181 estimators and at 0.066 yr^{-1} for the applicable estimators, slightly higher than the 0.05 currently
182 used in stock assessment, and a noticeable secondary peak around 0.2 yr^{-1} (Figure 2). Estimates
183 considered as applicable were distributed across the entire range of the plot and included the
184 highest point estimate, Jensen's third estimator at 0.322 yr^{-1} , indicating that technical
185 applicability does not need to imply plausible estimates. Likewise, estimates considered as non-
186 applicable covered a wide range, though narrower than the applicable ones, whilst the estimators
187 that can be used cautiously ranged from Alverson & Carney's estimator at the low end of the
188 range to the center of the range with Then's fifth estimator.

189 Using the different estimates in the statistical catch-at-age model for beaked redfish in the
190 Norwegian and Barents Seas, showed that a change in M , with otherwise the same settings and
191 input data has a strong effect on the SSB (Figure 3). With $M \geq 0.2$ for estimates across life-
192 history the model failed to converge at the standard maximum number of iterations (10^6), which
193 was the same for the weight-based estimators by Lorenzen as well as Jennings and Dulvy. When
194 doubling the number of iterations, the model did converge, but the resulting SSB-trajectories
195 were substantially different from all other SSB-trajectories, with the biomass always far below
196 the confidence interval. Only three estimators were completely within the confidence interval of
197 the benchmark run over the entire assessment period (1992 - 2016), ranging from $0.054 - 0.057$
198 yr^{-1} , all of them estimates across life-history (Table 2, Figure 3). Of the other estimates, most
199 across life-history, as well as age or size specific estimates tracked the benchmark trajectory until

200 the mid-2000s and then deviated outside the confidence interval (Figure 3). Estimators below
201 0.05 yr^{-1} were slightly below the benchmark trajectory, rising above the confidence interval in
202 the 2000s. Estimators above 0.05 yr^{-1} exhibited the opposite pattern, starting out slightly above
203 the benchmark SSB and then dropping below the confidence interval in about the same time
204 period. No estimators were outside the confidence interval at the beginning of the assessment
205 period and were within at the end and a small number of estimators, Then's sixth estimator,
206 Griffiths and Harrod's estimator as well as Rikhter and Efanov's second estimator, were outside
207 the confidence interval at both ends of the assessment period.

208 Age or size dependent estimators, exhibited no particularly different behaviour to those across
209 life history. Although it was excluded due to uncertainty about t_s , it is worth mentioning that
210 Chen and Watanabe's estimator yielded a SSB-trajectory within the confidence interval over the
211 entire assessment period. The length-based Gislason estimators were among those outside the
212 confidence interval at the beginning and end of the assessment period, although only very
213 slightly at the beginning (Figure 3). All the weight-based mortality estimates exhibited very high
214 M at small size and age, putting all but Charnov's estimator into the group that did not converge
215 at 10^6 iterations. Charnov's estimator was also among the cluster of estimates around the peak of
216 the frequency distribution, comprising otherwise Then's second estimator as well as Hamel's
217 first and second estimators (Table 2, Figures 2 & 3). However, whilst the three latter estimators
218 dipped below the confidence interval only in the mid-2000s, Charnov's estimator did so already
219 in the early 2000s.

220 **4 Discussion**

221 The wide variety of estimators examined yielded a wide range of SSB-trajectories, showing how
222 sensitive it is to different values of M . Another take home message is that age- or size-dependent
223 estimators, whilst more realistic, are no panacea and at least the weight-based ones exhibit very
224 high mortality rates at the lowest weights and therefore ages. Stepwise approaches like the
225 gnomonic approach, which also exhibits high mortalities at the lowest ages, or those used for
226 several Pacific rockfish stocks (Cope et al. 2016) may be a reasonable compromise. For the
227 assessment of beaked redfish in the Northeast arctic, short of adopting an age or size specific

228 mortality model, a higher natural mortality than is used now may be adopted, potentially 0.07 yr⁻¹
229 to reflect the frequency distribution of the examined, and applicable, estimators.

230 In the range of life histories exhibited by teleosts, *S. mentella* is closer to typical K-selectors than
231 to r-selectors, being a long-lived species (Campana et al. 1990), that matures late in life (Planque
232 and Nedreaas 2010) and produces comparatively few offspring (Zakharov et al. 1977; Lukmanov
233 1988; Drevetnyak and Gusev 1996). This combination of traits makes it difficult to manage in a
234 fisheries context, even more so as the fish undergo a change in habitat and lifestyle when
235 becoming mature (Anon. 2009). The majority of the mature stock migrates out of the Barents
236 Sea and adopts a pelagic lifestyle in the Norwegian Sea. This increases uncertainty in the
237 assessment as the mature component is only surveyed every third year and only in parts of its
238 habitat (ICES 2019). As the uncertainty caused by this ontogenetic migration cannot be avoided,
239 reducing uncertainty in the assessment model and its parameters is important. The reasoning
240 underlying the currently used $M = 0.05$ in the assessment model is derived from Hoenig's
241 estimator (Hoenig 1983) and expert judgement (Brodziak et al. 2011; Pope et al. 2021; ICES
242 2018a) and deserves to be re-evaluated with new approaches.

243 A wide range of taxa were used collectively in the development of the 48 considered M-
244 estimators, but the development of individual estimators was often based on a narrower range of
245 species. A number of estimators was derived from studying short-lived pelagic fishes, not
246 comparable to beaked redfish (Bayliff 1967; Sekharan 1974; Tanaka 1960; Alagaraja 1984). On
247 the other end of the spectrum there is Frisk's estimator (Frisk et al. 2001), developed specifically
248 for elasmobranch. Frisk's estimator provided a high estimate of 0.116 yr⁻¹, whilst those methods
249 developed from pelagic fish yielded estimates between 0.034 yr⁻¹ (Alagaraja 1984) and 0.085 yr⁻¹
250 (Bayliff 1967). Although these values are not very far off the value used in the assessment model
251 the underlying taxa made their use questionable.

252 Likewise, estimators based on snappers and groupers (Ralston 1987; Pauly and Binohlan 1996)
253 as well as on Mediterranean fish (Djabali et al. 1993, 1994) were not considered since they were
254 derived from taxa in substantially warmer climate zones, yielding higher mortality estimates,
255 averaging 0.11 yr⁻¹ across the estimators based on snappers and groupers, and 0.22 yr⁻¹ for
256 Djabali's estimator (Djabali et al. 1993, 1994).

257 Estimators with a broader basis appear to be more applicable, particularly when they afford to
258 select parameters for specific groups, such as Hoenig's estimator (Hoenig 1983) which offers to
259 use parameters specific for fish or the global dataset including whales and molluscs. Whilst this
260 yielded a low estimate of 0.036 yr^{-1} , other estimators based on many taxa resulted in a wide
261 range of estimates from 0.05 yr^{-1} from Gislason et al. (2010) to 0.17 yr^{-1} from Griffiths and
262 Harrod (2007).

263 A variable mortality rate is desirable for a long-lived species like beaked redfish. However,
264 mortality estimators that varied with age (Chen and Watanabe 1989), length (Gislason et al.
265 2010; Charnov et al. 2013) or weight (Peterson and Wroblewski 1984; Lorenzen 1996; Ursin
266 1967; Jennings and Dulvy 2008; Charnov et al. 2013) yielded highly variable results. With a
267 median value of 0.617 yr^{-1} the estimates from Jennings and Dulvy (2008) were the highest
268 estimated values and high values were consistently seen with all the weight dependent estimates,
269 except for Charnov's estimator (2013) which was more in line with other methods.

270 The gnomonic approach, which considers separate life stages (Caddy 1991, 1996; Torrejón-
271 Magallanes et al. 2021), is interesting for a species with several distinct life stages. However, the
272 method is geared for fish with high fecundity and a planktonic egg stage, whilst the genus
273 *Sebastes* is ovoviviparous, releasing comparatively few living larvae after they hatch within the
274 ovaries. Accordingly, the method produces extremely high estimates for early life stages of
275 beaked redfish and more sensible estimates for later stages.

276 Overall, the frequency distribution of the mortality estimators exhibited a cluster close to 0.07 yr^{-1}
277 ¹, most closely matched by Hamel's first and second as well as Then's second estimator ($0.067 -$
278 0.072 yr^{-1} , Hamel 2015; Then et al. 2015; Hamel and Cope 2022) and Charnov's weight-based
279 estimator (Charnov et al. 2013). All of these showed a similar development of the spawning
280 stock biomass when used in the assessment model, tracking the benchmarked SSB closely in the
281 early years of the assessment period and then gradually declining to values that fall short of the
282 accepted assessment (951 kt) by 231 kt to 320 kt. However, these values are even in the worst
283 case more than 300 kt above the precautionary biomass (B_{pa}) of 315 kt for this species.

284 The Atlantic ocean hosts only four species of redfish and only three are of commercial interest
285 and assessed, *Sebastes norvegicus*, *S. mentella* and *Sebastes fasciatus*. There appears to be a

286 consensus for the Atlantic species in using a constant M across all age classes, ranging between
287 0.05 yr^{-1} and 0.1 yr^{-1} (Miller et al. 2008; ICES 2021, 2022). Compared to the Atlantic the
288 diversity of mortality estimates used for *Sebastes* species along the Pacific coast of North
289 America is much higher, with several species assessed as complexes (Dick et al. 2017, 2021).
290 Here, natural mortality often takes different values for males and females (Cope et al. 2016; Dick
291 et al. 2021) and in several cases with a step-change or a linear increase over a short period of
292 time between younger and older ages (Cope et al. 2016). Several species do not have a direct
293 input of M , but use a prior distribution derived with the natural Mortality Tool (NMT, Cope and
294 Hamel 2022), often using the log-normal Hamel prior (Dick et al. 2021; Monk et al. 2021;
295 Spencer and Ianelli 2021). Another common approach is based on T_{max} using Hoenig's approach
296 (Hoenig 1983; Dick et al. 2017) or Then's revision thereof (Then's third estimator, Then et al.
297 2015). For squarespot rockfish (*Sebastes hopkinsi*) off California a similar method as in the
298 present study was used, taking the median of $M = 0.133$ of four empirical estimators (Cope et al.
299 2021). This value is also typical for Pacific species, where estimates are overall higher than for
300 their Atlantic congeners. However, a recent re-evaluation for the *S. mentella* and *S. fasciatus*
301 complex in the northwestern Atlantic found a median prior of 0.125 yr^{-1} (Cadigan et al. 2022), in
302 line with estimates for Pacific species. Contrarily, as demonstrated for yelloweye rockfish
303 (*Sebastes ruberrimus*) with a median prior of 0.05 yr^{-1} , Pacific species may also exhibit low
304 estimates of natural mortality (Cope and Hamel 2022).

305 One of the limitations of the current SCA used for *S. mentella* in the Northeast Arctic is that,
306 whilst being technically able to, in practice it cannot estimate F and M in the same model run.
307 Therefore, estimating a prior with uncertainties, as in Cadigan et al. (2022) and Cope and
308 Hamel (2022), may not currently be useful for the assessment. However, as this study
309 demonstrates, relying on any single estimator may be misleading and before examining a range
310 of estimators one has to carefully curate the candidates in relation to what taxa they are based on
311 and whether these taxa occupy similar habitats than the species one wants to derive an estimate
312 of M for. Age or size dependent estimates are attractive as they account for changes in life
313 history. However, they may be difficult to apply to very early ages. In case of the gnomonic
314 estimator (Caddy 1991, 1996; Torrejón-Magallanes, Morales-Bojórquez, and Arreguín-Sánchez
315 2021) this may be due to it being developed with more typically commercially exploited species,
316 with an egg stage, in mind. Adaption to ovoviviparous and viviparous species may make it more

317 suitable for the genus *Sebastes*. A large number of estimators rooted in ecological theory was
318 evaluated as useful “with caveats.” Although they may be more widely applicable than those
319 based on specific taxa, they may be more useful when used with a species- or genus-specific
320 adjustment factor, similar to the adjustment factor for *Clupeidae* in Pauly’s first estimator (Pauly
321 1980).

322 Even after all these considerations there remain some estimators yielding implausibly high
323 estimates for M , whilst being technically applicable. As a simple plausibility test estimated
324 values for M can be used to determine the corresponding longevity, using any longevity-based
325 estimator, such as Hamel’s first estimator (Hamel 2015; Hamel and Cope 2022). Using this
326 estimator under the consideration that the bulk of the population of *S. mentella* in the Norwegian
327 and Barents Seas ranges between age-11 and age-40 (Planque and Nedreaas 2010), any estimate
328 for M above 0.135 yr^{-1} may be considered implausibly high. Such simple reality checks can
329 support sensitivity analysis, recommended by Maunder et al. (2023) as part of good practices for
330 using M in stock assessments. The same paper reviewed several estimators also considered here,
331 partially coming to different conclusions based on the criteria these authors used (Maunder et
332 al. 2023).

333 In summary, taking the peak of the frequency distribution or the median across a range of pre-
334 screened estimators may be the most recommendable way forward if using a single mortality
335 estimate. However, age and size specific estimates may be preferable, provided they can be
336 adjusted for earlier ages and lower weights. Step-changes may be a useful compromise,
337 particularly for stocks like *S. mentella* in the Northeast Arctic with well defined life stages and
338 corresponding habitats.

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344

345 **Table and figure captions**

346 **Table 1:** Parameters used in the equations for the different mortality estimators. Apart from the
347 parameters shown here, latin letters are used to denote coefficients and exponents in the
348 equations presented in the supplementary material.

349 **Table 2:** Natural mortality estimates from the 48 considered estimators and their applicability to
350 beaked redfish (*Sebastes mentella*). Estimators marked with an asterisk are age, length or weight
351 specific and the estimate given is the median of the considered range (only mature and immature
352 adults for the gnomonic approach). Estimators evaluated as *with caveats* may be applicable but
353 suffer from reliability issues or make assumptions that are difficult to assess.

354 **Figure 1:** Von Bertalanffy growth functions calculated with growth parameters from 3 different
355 sources. Aanes (2012) assumes $t_0 = 0$. Nedreeaas (1990) covers the most complete range of
356 beaked redfish life history as it includes the juvenile stage, whilst the DeepFishMan-report
357 (2010) covers only the adult population.

358 **Figure 2:** Frequency plot of point estimates for natural mortality rates. Estimators that have been
359 evaluated as applicable to beaked redfish are indicated with green ticks and labels, cautiously
360 useful estimators are orange and those deemed not applicable are red. The dashed red line
361 indicates the value 0.05 yr^{-1} currently used in the assessment, the blue dashed line the peak of the
362 frequency distribution for all estimators (0.065 yr^{-1}) and the green dashed line the peak for the
363 applicable estimators (0.066 yr^{-1}).

364 **Figure 3:** SSB from the benchmark assessment model with selected estimates of natural
365 mortality demonstrating the different trajectories taken by SSB when using higher or lower
366 estimators as well as constant estimators or estimators depending on age or size. Only a few
367 estimators yielded results that were in the currently accepted confidence interval (shaded area)
368 across the assessment period, indicated by bold font in the legend. The thick red line indicates a
369 M -value of 0.07 yr^{-1} which would be a proposed new value to adopt for stock assessment should
370 a constant value for M be retained.

371

372 **Tables**

373 **Table 1**

374 *Table 4.1: Parameters used in the equations for the different mortality estimators. Apart from the*
 375 *parameters shown here, latin letters are used to denote coefficients and exponents in the*
 376 *equations presented in the supplementary material.*

Symbol	Parameter
α	Parameter of the length-weight relationship
α_G	Proportionality constant of the gnomonic approach
β	Exponent of the length-weight relationship
δ_i	duration of an individual gnomonic interval
θ_i	annual proportional duration of a gnomonic interval
τ	Mean environmental temperature
F	Fishing mortality
G	Proportion of overall natural mortality rate
K	Parameter of the von Bertalanffy growth curve
L_∞	Asymptotic fish length
l	Individual fish length
l_m	Length at reproductive maturity
M	Natural mortality rate
M_i	Natural mortality rate of a gnomonic interval
M_l	Natural mortality rate at length l
M_t	Natural mortality rate at age t
M_w	Natural mortality rate at weight w
MLF	Mean Lifetime Fecundity
N_i	Number of individuals in a population at time or age i

Symbol	Parameter
$N_{i,j}$	Number of individuals in year-class j at time i
N_0	Number of individuals in a year class at age zero
n	Number of individuals in a sample
n_e	Effective sample size, as defined for use in Kenchington's Estimator
P	Proportion of fish in a population surviving to age T_{max}
T_{max}	Maximum age, observed or assumed
T_{∞}	Age at which fish are expected to grow to a length of L_{∞} -5 mm
$T_{95\%}$	Age at which fish are expected to grow to $0.95 * L_{\infty}$
t	Water temperature in °C
t_a	Average female adult lifespan
t_c	Youngest age fully represented in the catch or the minimum age considered in n_e
t_m	Age at reproductive maturity (at 50% mature)
t_{mb}	Age at which year-class achieves its maximum biomass in the absence of fishing
t_s	Age at onset of senescence
t_0	Parameter of the von Bertalanffy growth curve
W_{α}	Weight at first reproduction
W_{∞}	Asymptotic fish weight
w	Fish weight
Z	Total mortality rate

378 **Table 2**

379 *Table 4.2: Natural mortality estimates from the 48 considered estimators and their applicability*
 380 *to beaked redfish (*Sebastes mentella*). Estimators marked with an asterisk are age, length or*
 381 *weight specific and the estimate given is the median of the considered range (only mature and*
 382 *immature adults for the gnomonic approach). Estimators evaluated as with caveats may be*
 383 *applicable but suffer from reliability issues or make assumptions that are difficult to assess.*

Estimator	Category	Parameters	Estimate	Evaluation
Bayliff's	age	T_{max}	0.085	not applicable
Then's I	age	T_{max}	0.068	not applicable
Hoenig's	age	T_{max}	0.036	applicable
Hamel's I	age	T_{max}	0.072	applicable
Then's II	age	T_{max}	0.071	applicable
Then's III	age	T_{max}	0.094	with caveats
Sekharan's	age	P, T_{max}	0.061	not applicable
Tanaka's	age	P, T_{max}	0.040	not applicable
Kenchington's	age	T_{max}	0.065	not applicable
Charnov & Berrigan's	age	t_m	0.200	applicable
Jensen's I	age	t_m	0.150	not applicable
Alverson & Carney's	age	K, t_{mb}	0.014	with caveats
Zhang & Megrey's	age	β, K, t_0, t_{mb}	0.019	applicable

Estimator	Category	Parameters	Estimate	Evaluation
Roff's I	age	K, t_m	0.213	not applicable
Rikhter & Efanov's I	age	β, K, t_0, t_m	0.057	applicable
Rikhter & Efanov's II	age	t_m	0.116	applicable
Chen & Watanabe's*	age	K, t, t_0	0.051	not applicable
Alagaraja's	life history correlates	K, t_0, L_∞	0.034	not applicable
Ralston's I	life history correlates	K	0.042	not applicable
Then's V	life history correlates	K	0.165	with caveats
Ralston's II	life history correlates	K	0.177	not applicable
Pauly's I	life history correlates	K, L_∞, τ	0.115	applicable
Pauly's II	life history correlates	K, L_∞, τ	0.101	not applicable
Jensen's III	life history correlates	K, τ	0.322	applicable
Griffiths & Harrod's	life history correlates	K, W_∞	0.167	applicable
Then's VI	life history correlates	K, L_∞	0.118	applicable
Djabali's	life history correlates	K, W_∞	0.220	not applicable

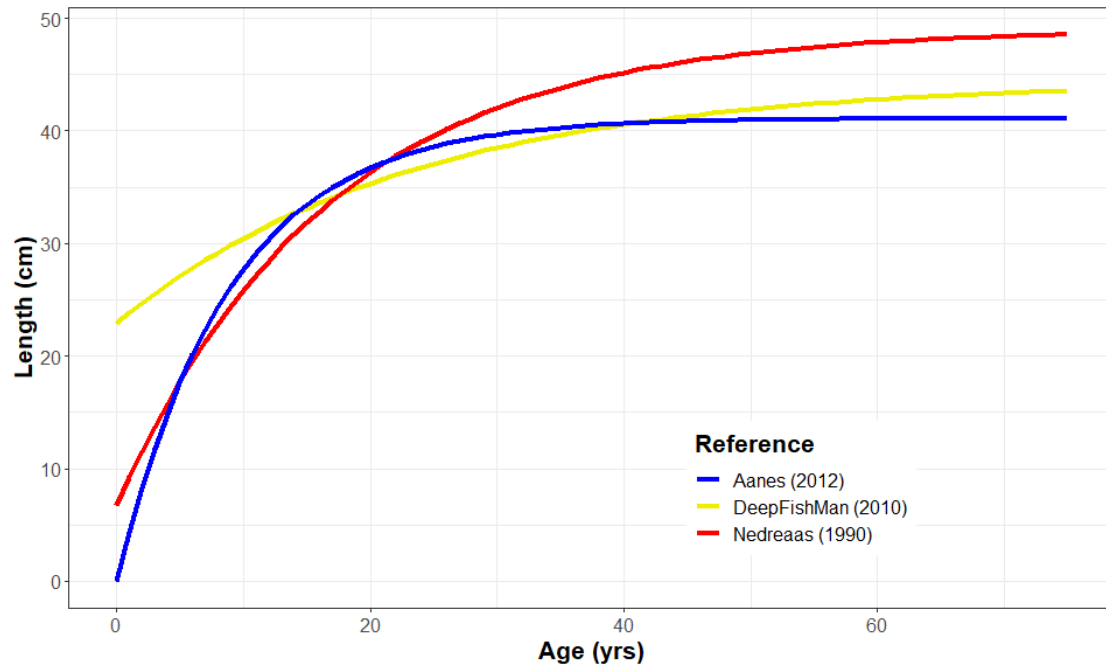
Estimator	Category	Parameters	Estimate	Evaluation
Frisk's	life history correlates	K	0.116	not applicable
Gislason's I*	life history correlates	K, l, L_∞	0.051	applicable
Gislason's II*	life history correlates	K, l, L_∞	0.055	applicable
Cubillos'	life history correlates	K, t_0	0.079	with caveats
Gunderson's	life history correlates	GSI	0.054	applicable
Hamel's III	life history correlates	GSI	0.055	applicable
Mangel's	life history correlates	K, t_m	0.259	applicable
Jensen's II	ecological theory	K	0.065	with caveats
Then's IV	ecological theory	K	0.073	not applicable
Hamel's II	ecological theory	K	0.067	applicable
Peterson & Wroblewski's*	ecological theory	w	0.244	not applicable
Lorenzen's*	ecological theory	w	0.445	applicable
Ursin's*	ecological theory	w	0.110	with caveats
Jennings & Dulvy's*	ecological theory	w, t	0.617	applicable
Roff's II	ecological theory	K, L_∞, l_m	0.056	with caveats
Groeneveld's	ecological theory	L_∞, l_m	0.204	not applicable
Charnov's*	ecological theory	w, W_α	0.058	applicable

Estimator	Category	Parameters	Estimate	Evaluation
Gnomonic determinate simple*	ecological theory	$\alpha_G, \delta_i, \theta_i, G, M_i,$ <i>MLF</i>	0.131	with caveats
Gnomonic determinate extended*	ecological theory	$\alpha_G, \delta_i, \theta_i, G, M_i,$ <i>MLF</i>	0.216	with caveats
Gnomonic stochastic simple*	ecological theory	$\alpha_G, \delta_i, \theta_i, G, M_i,$ <i>MLF</i>	0.132	with caveats
Gnomonic stochastic extended*	ecological theory	$\alpha_G, \delta_i, \theta_i, G, M_i,$ <i>MLF</i>	0.218	with caveats

384

385 **Figures**

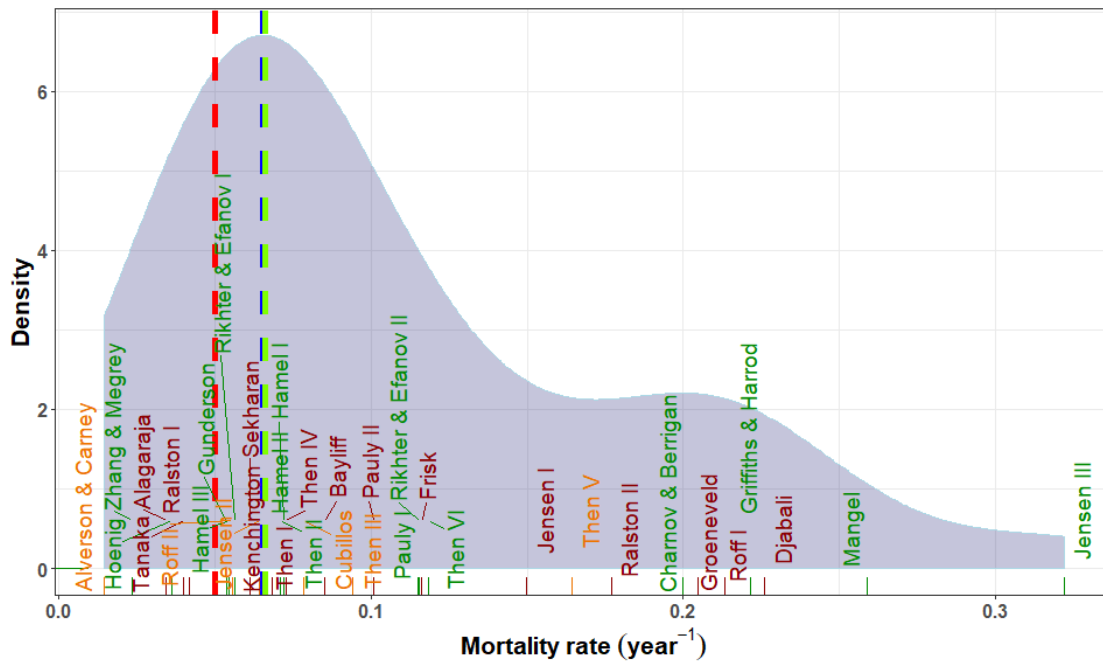
386 **Figure 1**



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388

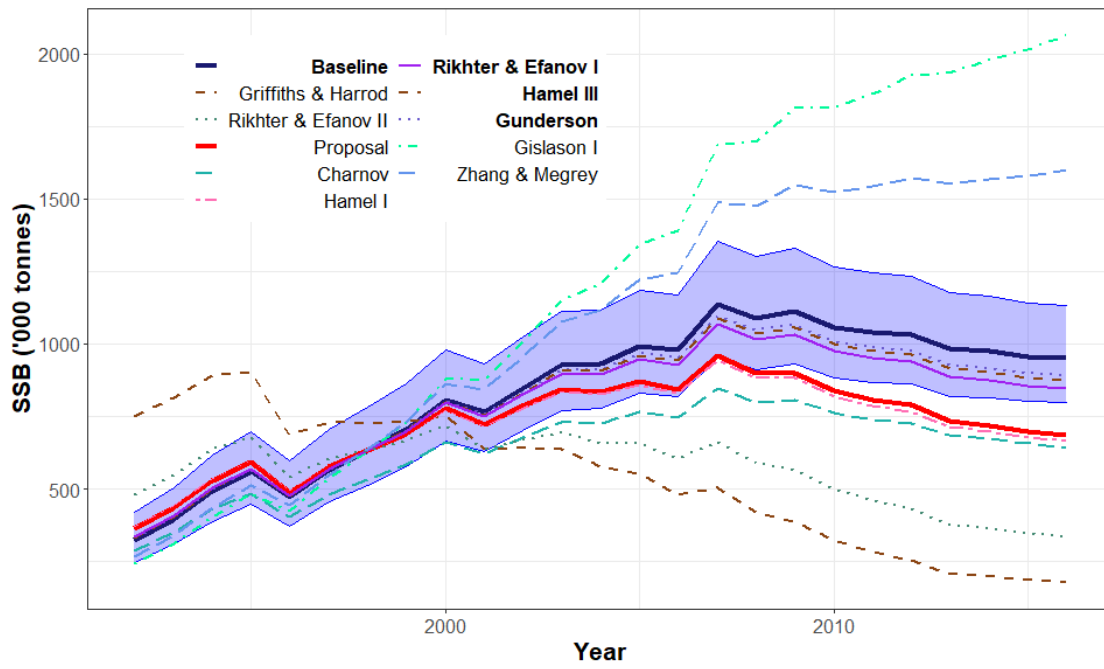
389 **Figure 2**



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392 **Figure 3**



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