

## Estimating $g(0)$ from single observer data

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### ABSTRACT

In this paper we estimate  $g(0)$ , or equivalently the effective strip half-width, using data from a line transect survey which has been operated in double observer mode only a small fraction of the time. By letting the proportion of double observer effort approach zero, by increasingly masking data from one of the observers, we find that the estimate of  $g(0)$  does not break down. This conclusion rests on the fact that we are using both forward and perpendicular distances, and is perhaps only relevant to northeastern Atlantic minke whale surveys.

KEYWORDS: LINE TRANSECT SURVEYS, INDEPENDENT OBSERVERS, HAZARD PROBABILITY MODEL, MINKE WHALES

### INTRODUCTION

Line transect surveys with two independent observers are commonly used to estimate the abundance of whales within the context of the IWC. The main purpose of the independent-observer design is to allow estimation of the detection probability on the track line,  $g(0)$ . It is well known that  $g(0)$  is an unidentifiable parameter in the classical one-observer design, when only the perpendicular distances ( $x$ ) are recorded. However, if one instead records both radial distance ( $r$ ) and sighting angle ( $\theta$ ) for each sighted whale this conclusion no longer holds true. In the present paper we claim that  $g(0)$  can be estimated when single-observer  $(r, \theta)$  data are integrated with a model for the diving behaviour of the whales, i.e. using a hazard probability model. We do not provide a solid proof of this claim, but rather empirical evidence about what happens when the proportion of double observer effort goes to zero. This is achieved by starting out with an ordinary double observer survey, and then masking data from one of the observers to an increasing degree.

The hazard probability model is designed as a model for line transect data where the target has discrete availability, such as with diving whales (Schweder 1974). The model can accommodate both the situation that  $g(0) < 1$  and  $g(0) = 1$ . Over the last decade it has been applied to Japanese and Norwegian minke whale line transect data (Okamura *et al.* 2003; Schweder *et al.* 1996; Schweder *et al.* 1997; Skaug *et al.* 2004b). These analyses have used  $(r, \theta)$  as the datum, but the hazard probability model has also been employed to perpendicular distances ( $x$ ) only (Skaug & Schweder 1999). So far nobody has fitted the model to single observer data. In the present paper we partly do so by masking data from one of the observer platforms in the minke whale double platform surveys conducted in the Northeastern Atlantic during the two periods 1996-2001 (Skaug *et al.* 2004a) and 2002-2007 (Bøthun *et al.* 2009). The results are then compared to the results obtained by using data from both platforms.

## **MATERIAL AND METHODS**

### **Hazard probability model for independent observers**

Consider first a single observer with hazard probability function  $Q(x, y)$ . The detection function, i.e. the probability of detecting a whale which is present at perpendicular distance  $x$ , is given as

$$g(x) = 1 - \exp\left(-\frac{\alpha}{v} \int_0^\infty Q(x, y) dy\right),$$

where  $\alpha$  is the surfacing rate of the whale, and  $v$  is the speed of the observer. The probability density of the relative position  $(x, y)$  of the initial observations is given as

$$f(x, y) = \frac{\alpha}{vw} Q(x, y) \exp\left(-\frac{\alpha}{v} \int_y^\infty Q(x, u) du\right), \quad 0 \leq x \leq W, \quad y \geq 0,$$

where  $w$  is the effective strip half-width given by

$$w = \int_0^W g(x) dx = \int_0^W \left[1 - \exp\left\{-\frac{\alpha}{v} \int_0^\infty Q(x, y) dy\right\}\right] dx. \quad (1)$$

There are typically two or more independent observers (or observer platforms). In the minke whale surveys in the Northeastern Atlantic a symmetric two-platforms design is used (Skaug *et al.* 2004), while in the Antarctic three platforms, with a partly asymmetrical configuration, have been used (Okamura *et al.* 2003). The combined observer  $A \cup B$ , i.e. viewing  $A$  and  $B$  as being a team, has hazard probability function

$$Q_{A \cup B} = Q_A + Q_B - Q_A Q_B = 2Q - Q^2.$$

To get expressions for  $g(x)$ ,  $f(x, y)$  and  $w$  for the combined observer  $A \cup B$  we can directly insert  $Q_{A \cup B}$  in the above formulae. Further, each animal detected by  $A \cup B$  sets up an experiment with trinomial outcome  $u \in \{A, B, AB\}$ . Conditionally on the position  $(x, y)$  the probability distribution of  $u$  is

$$q(u | x, y) = \{Q_{A \cup B}(x, y)\}^{-1} \begin{cases} Q_A(x, y)\{1 - Q_B(x, y)\}, & u = A; \\ Q_B(x, y)\{1 - Q_A(x, y)\}, & u = B; \\ Q_A(x, y)Q_B(x, y), & u = AB. \end{cases}$$

### Experimental setup

Data from two survey periods (1996-2001 and 2002-2007) of the Norwegian minke whale surveys were used. The first survey period involved 870 initial sightings made by  $A \cup B$ , and 623 Bernoulli trials, while in the second survey period there were 633 initial sightings and

616 Bernoulli trials. The best fitting covariate model and bias correction factor obtained in previous studies (Bøthun *et al.* 2009; Skaug *et al.* 2004a) were employed. All parameters of the hazard probability model, except the parameter distinguishing platform A and B (the platform B effect), were estimated by maximum likelihood. The “platform B effect” was fixed to the value obtained in previous studies (Bøthun *et al.* 2009; Skaug *et al.* 2004a) in order to avoid identifiability issues when the amount of data from platform B becomes small. The software package AD Model Builder (ADMB-Project 2009) was used to maximize the likelihood function.

The model was fitted to reduced datasets, obtained by masking information from one of the platforms, B say, for a proportion ( $q$ ) of the initial sightings. This is intended to emulate a situation where a part  $q$  of the survey was run in single platform mode (only A active), while the remaining part  $1-q$  was run in double platform mode. Note that  $q$  applies to the number of sightings, not survey time, although our assumption is that these are roughly equivalent. Hence, for each  $A \cup B$  sighting in the dataset a decision is made whether or not B is being masked. What happens to the sighting, and associated Bernoulli trials, when B is masked depends on whether the initial sighting of this whale was made by A, B or AB (see Table 1). Masking B may involve turning a Bernoulli trial for A into an initial sighting (individual “01” in Table 1). The selection of the  $A \cup B$  sightings to which B-masking was applied was made at random. This procedure was repeated up to 50 times for each value of  $q$ , (letting  $q$  range between 0 and 1). For each of these 50 datasets parameters were estimated by maximum likelihood, and the across-dataset variability of  $w_A$  was assessed (see below). In exactly the same manner, datasets were generated with platform A being masked instead of B.

For comparison reduced datasets with both A and B being masked for a proportion  $q$  of the sightings were also generated. The latter emulates the effect of reducing the total survey effort (both platforms). In this case masking worked as follows. From the pool of  $A \cup B$  sightings a fraction  $q$  was chosen at random, and removed from the dataset. Note that this step is involved also when masking only one platform, and exactly the same random subset was used as when masking A or B only. It is important to note that “masking both A and B”, may involve removing sightings made by only one platform.

The parameter of interest in the study was the coefficient of variation (CV) of  $w_A$ , i.e. the effective strip half-width of platform A. The median (over the 50 simulated datasets) of the estimated CV's (based on the observed Fisher information)

## **RESULTS AND DISCUSSION**

The CV of the estimated  $w_A$  increases as the proportion of the dataset being masked increases (Figure 1), as is to be expected. This holds true regardless of which platform is being masked. Masking both A and B in a period yields a higher CV than masking only a single platform. The only exception to this occurred at masking level 99% for the 2002-2007 dataset, where masking data from platform A apparently increases the CV (black versus white bar). It should, however, be noted that removing 99% of the initial observations for both platforms is an extreme situation, which actually leads to convergence problems for AD Model Builder, in the sense that the Hessian matrix corresponding to the maximum likelihood estimate is not positive definite. For this reason, the 95% and 99% masking levels yielded only 49 and 43

replications, respectively, and not the target of 50 simulation replica, which was obtained for the other cases.

As shown in Figure 1, there is no dramatic bias in  $w_A$  occurring for masking levels less than 80%. The figure shows the mean value of  $w_A$  calculated across the 50 simulation replica.

In conclusion the estimate of  $w_A$  does not break down as the masking fraction goes to 1, i.e. we approach a situation with only a single platform. This is especially true when platform B is being masked.

## CONCLUSION

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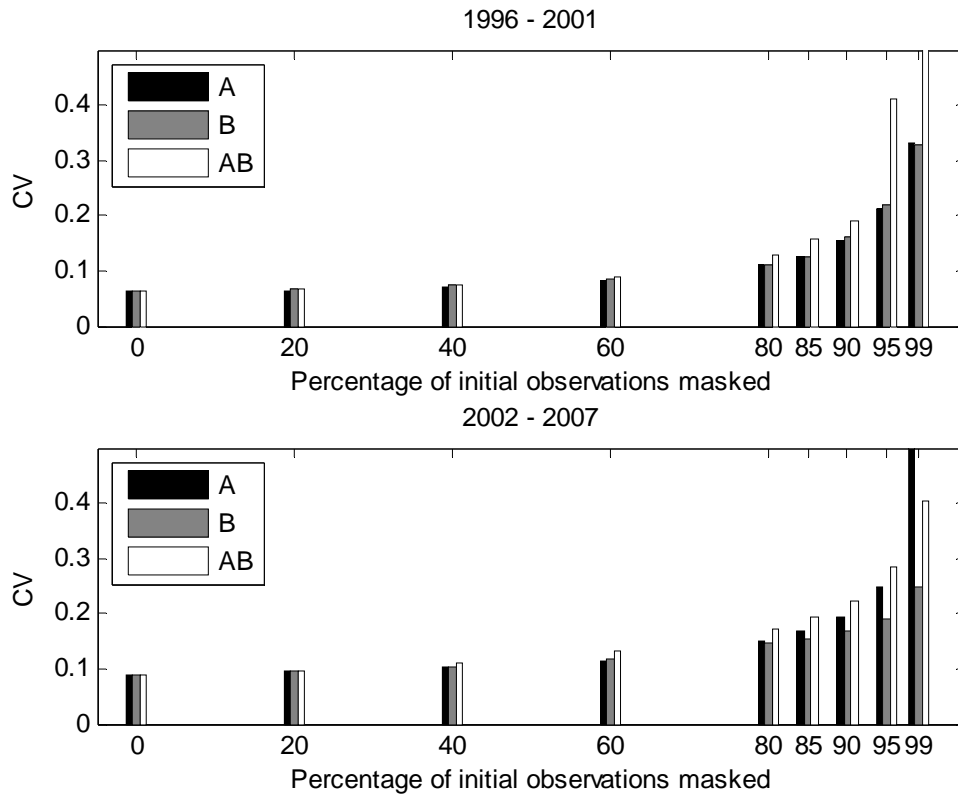
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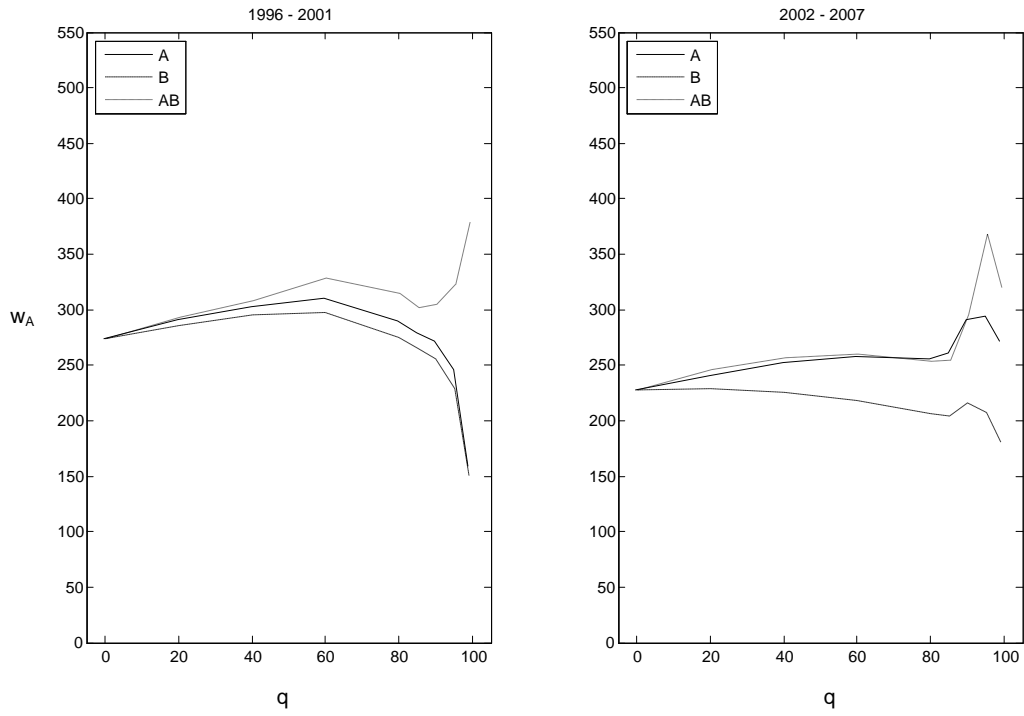
ID-A	ID-B	Initial	Bernoulli trial	
			On tri.	Outcome
<u>Original data</u>				
	00.1	B		
	00.2		A	Failure
00.1	00.3		A	Success
01.1	01.1	AB		
	02.1	B		
<u>Data when B has been masked</u>				
00.1		A		
01.1		A		

**Table 1:** Example of a small dataset before and after masking platform B. The first two columns show the surfacing identification number on the form whale. Surfacing have been aligned (duplicate identification) correctly across the two platforms. The two first columns shows the whale identification number as recorded by platform A and B individually, in the format “n.m”, where n is a two digit whale-id and m is a one digit surfacing-number. The column “Initial” shows which platform made the initial observation of this individual, while “Bernoulli trial” shows which platform is on trial and the outcome of the trials for each subsequent surfacing. Note that after B has been masked there can be no Bernoulli trials.



**Figure 1** Coefficient of variation (CV) of the estimated  $w_A$  against masking level  $q$  (proportion of sightings masked). The shading indicates which platform is being masked.





**Figur 2** Average  $w_A$  against masking level  $q$  (proportion of sightings masked).