Linearity of fisheries acoustics, with addition theorems

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An experiment to verify the basic linearity of fisheries acoustics is described. Herring (*Clupea harengus* L.) was the subject fish. Acoustic measurements consisted of the echo energy from aggregations of encaged but otherwise free-swimming fish, and the target strength functions of similar, anesthetized specimens. Periodic photographic observation of the encaged fish allowed characterization of their behavior through associated spatial and orientation distributions. The fish biology and hydrography were also measured. Computations of the echo energy from encaged aggregations, derived by exercising the linear theory with the target strength functions of anesthetized fish and gross behavioral characteristics of encaged fish, agreed well with observation. This success was obtained for each of four independent echo sounders operating at frequencies from 38 to 120 kHz and at power levels from 35 W to nearly 1 kW. In addition to demonstrating the basic linearity of fisheries acoustics, the experiment verified both conventional acoustic measurements on anesthetized fish, at least for averaging purposes, and the echo integration method. Two simple theorems summarizing the meaning of linearity for use with the echo integration method are stated.

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INTRODUCTION

Assessment of fish stocks by means of the echo integration method demands detailed knowledge about the backscattering cross section or target strength of fish.[1] A popular means of acquiring such information is by direct measurement on anesthetized, stunned, or killed specimens.[2,3] While such measurements allow a high degree of control, the extent to which they are representative of free-swimming fish in the wild is unknown.

It was to resolve this matter for the important class of swimbladder-bearing fish that the present investigation was undertaken. In particular, it was hoped that the connection between measurements on anesthetized fish and measurements on encaged but otherwise free-swimming fish could be established unambiguously. Thus, recognizing that the two prominent extrinsic dependences of fish target strength are the orientation and depth-or depth-history-related state of the swimbladder, it was apparent at the outset that the one effect must be isolated from the other.

Given the distinguished history of attempts to elucidate depth-induced effects on the target strength,[4–6] which are still unclear, it was decided to avoid depth effects entirely by conducting all measurements near the surface, in the manner of Röttingen[7] and Nakken and Olsen.[3] Transferring fish from pens to the tilting suspension or net cage could then be accomplished swiftly, and the acoustic measurements commenced immediately upon positioning the fish or net cage in the center of the transducer beam.

Naturally, the measurements would have to be made ventrally; but as the purpose of the experiment was verification of a methodology, and not derivation of target strengths to be applied directly to field measurements, this was no drawback. In fact, the configuration of ventrally executed measurements had everything to recommend it—from the principal advantage of being able to maintain the subject fish near the surface at all times, to the very practical advantage of precluding bubble entrapment by the transducers or their housings due to disturbances beneath them. In addition, if the ventral aspect measurements on the anesthetized fish were found to be representative of the encaged, free-swimming fish, then accompanying measurements of the dorsal aspect function could presumably be applied in survey work, given sufficient knowledge about the circumstances of fish occurrence.

For the sake of redundancy, the measurements were to be performed on each of two species, with each of four echo sounders operating at frequencies from 38–120 kHz and at power levels spanning a wide range. A large number of data were to be collected to establish possible forthcoming results with a high degree of confidence. In the event, the redundant design proved its worth, and useful data were collected in abundance.

Although the original major objective was verification of the target strength functions of anesthetized fish, it was discovered early that the linearity of the whole acoustical process would be tested. Success with this would also enable the basic echo integration method to be verified. Thus the theme of the work became establishment of the linearity of fisheries acoustics. In this, conventional measurements of target strength functions provide the fundamental acoustical knowledge about fish. In addition, the echo integration method is one of the consequences of linearity.

The plan of the paper is the following: presentation of the simple linear theory for acoustic scattering by fish aggregations, statement of the problem of verification, outline of an experimental design, description of materials and method, including data analysis and results, discussion of these, and listing of summary conclusions.

I. THEORY

According to the hypothesis of linearity, the acoustic echo from an aggregation of fish is merely the sum of the individual echoes.[1,8–13] If the process of reception is linear, then the equivalent received pressure field $p_{\rm rec}$ is just

$$p_{\rm rec} = \sum_{i=1}^{n} p_{{\rm rec},i}\,, \tag{1}$$

where $p_{{\rm rec},i}$ is the component due to the ith fish of n. In terms of the backscattering cross section σ, product of transmit and receive beam patterns b^2, and cumulative gain G, including reference pressure level of the source, receiver amplification, and possible time-varied gain (TVG),

$$p_{{\rm rec},i} = (Gb^2\sigma)_i^{1/2} s_i, \tag{2}$$

where s_i is the echo waveform, which is generally different from that of the ensonifying signal. The several factors in Eq. (2) are generally implicit or explicit functions of fish orientation and position in the beams of the acoustic source and receiver, not to mention physical state of the fish.

Compounding of the received echoes from individual fish by Eq. (1), squaring, and integrating in time, yields the well-known expression for the echo energy ϵ; namely,

$$\epsilon = \sum_{i=1}^{n}\sum_{j=1}^{n} (Gb^2\sigma)_i^{1/2}(Gb^2\sigma)_j^{1/2} c_{ij}, \tag{3}$$

where c_{ij} is the correlation coefficient of echo waveforms from the (i,j) pair of fish,

$$c_{ij} = \frac{2}{T}\int_{-\infty}^{\infty} s_i\left(t-\frac{2r_i}{c}\right) s_j\left(t-\frac{2r_j}{c}\right) dt, \tag{4}$$

where T is the duration of the transmit signal, t is the time, r_i is the range of the ith fish, and c is the speed of sound. The factor G in Eq. (3) has been scaled by incorporation of several multiplicative constants so that ϵ has the units of energy.

Statistical evaluation of Eq. (3) for ordinary sonar signals is straightforward. In the mean of a large number of observations and in the absence of noise, assumed implicitly above,

$$\text{Av}\,\epsilon = n_0\langle Gb^2\sigma\rangle, \tag{5}$$

where n_0 is the average number of fish detected per ping and $\langle Gb^2\sigma\rangle$ is the ensemble average of $Gb^2\sigma$. This is determined from the general distributional characteristics of the fish. In terms of the cumulative distribution function F,

$$\langle Gb^2\sigma\rangle = \int (Gb^2\sigma)_{l,\beta}\, dF, \tag{6}$$

where the subscripts attached to the integrand denote the length l of the fish and other biological characteristics β, such as species, condition when observed acoustically, and behavior insofar as social interactions may influence the fish as acoustic scatterers. The probability element dF shares these described dependences together with the suppressed position and orientation dependences of the fish when being observed.

Higher-order moments of the echo energy can be computed. These are important for understanding the nature of variations in observations of fish aggregations, but do not, in themselves, influence the mean value. Since it is the correctness of this first-order moment, as expressed by Eq. (5), which determines the success or failure of the echo integration technique, further statistical development of Eq. (3) is unnecessary here.

II. PROBLEM OF VERIFICATION

The gist of linearity in fisheries acoustics is expressed most succinctly in Eq. (5): given a sufficient number of acoustic observations on a fish aggregation, the mean density of sensed fish, or mean number per ensonification, can be estimated without bias. This consequence of linearity is a tenet of the echo integration method of estimating fish density, hence may deserve closer examination.

There is a mass of powerful, circumstantial evidence for the truth of Eq. (5). This lies in the early observations of Truskanov and Scherbino,[14] in many measurements of encaged fish aggregations,[7] and in consistent, long-term successes with the echo integration method.[15] *A priori* support is derived from well-known and oft-confirmed acoustical and electromagnetic theories for echo formation by random collections of scatterers,[16] which have been traditionally accepted in fisheries acoustics.[17,18]

What hard evidence is there, however, for the truth of the equation, hence that of the echo integration method? In fact, what could constitute a proof or convincing demonstration of either, given the nearly mutually exclusive requirements for acoustically clean measurements on a fish aggregation and exact knowledge about these fish during their measurement? This is the problem of verification.

In order to verify the echo integration method as represented by Eq. (5), it must be possible to specify each term of the equation for the same conditions of observation of the same fish aggregation. The constituents of this specification are the following: measurement of the echo energy ϵ from an aggregation of known number density n_0, determination of the cumulative gain G of the receiver and coupled echo integrator and of cumulative patterns b^2 of the transmitter and receiver, simultaneous observation or determination of the behavior of the encaged fish, i.e., of their collective states of orientation and position, and independent knowledge of the backscattering cross section σ of the aggregating fish.

To be convincing, these data must be gathered on a fish aggregation under nontrivial circumstances. Thus the aggregation density should be sufficiently high, or the duration of the ensonifying signal should be correspondingly long, so that fish echoes overlap and the correlation coefficient of Eq. (4) is not identically zero for all pairs of fish. Similarly, the ensonification frequency should be sufficiently high so that the phases of the overlapping echoes are not all identical, which would be equivalent to a unity correlation coefficient, another tautological situation. The frequency should also be sufficiently high so that echoes from individual fish are sensitive to their orientation. Within these limits, the potential complexity is great. This may incidentally explain why the only echo verification experiment considered by Swingler and Hampton,[19] in a refutation, involved tethered spherical polystyrene floats.

III. AN EXPERIMENT

Given the desire to verify the echo integration method in a nontrivial manner, consistent with the above requirements, but also as simply as possible, a series of experiments on encaged fish was performed in the summer of 1980. As noted in the Introduction, the original motivation was a verification of the conventional method of determining fish target strength functions by measurement on anesthetized or stunned fish, tethered and tilted about a fixed position in the beam of an echo sounder.[2,3] This objective was supplanted, however, by the larger, more encompassing goal of verification of the basic linearity of fisheries acoustics.

In essence, the experiment consisted in simultaneous acoustic and photographic measurements of an encaged aggregation of otherwise free-swimming fish. The least considered density was sufficient for the net cage geometry and pulse duration to ensure 50% overlapping of fish echoes. The acoustic wavelength corresponding to the least ensonification frequency was less than the dimensions of dominant scattering features of individual fish, ensuring both the variable nature of the correlation coefficient between overlapping echoes and the very sensitive orientation dependence of the fish backscattering cross section.

To keep other variables of the measurement process as simple as possible, the beamwidths of the several transducers were required to be broad with respect to the transverse dimension of the net cage, yet narrow enough to permit placement of an underwater television camera near the net cage, in the acoustic shadow region between the main and first side lobes. The relative broadness of the beam facilitated collection of data with good statistics, since arbitrariness in the spatial distribution of fish within the net cage could not in itself produce large variations. Problems of the kind experienced in the narrow-beam measurement of encaged fish, cf. Refs. 20–23, for example, could thus be circumvented. Tailoring of the transducer beamwidths also facilitated acoustic hiding of the television camera, allowing simultaneous photography and acoustic measurement, hence determination of the spatial and orientation distributions of the fish during their acoustic measurement.

Measurement of single-fish target strength functions was performed immediately before or after each series of encaged fish measurements. Thus, were the acoustic properties of the fish to change over long periods of time, this could not prejudice the ultimate comparison of observed and computed effective backscattering strengths derived in testing Eq. (5). Short-term variations in the acoustic properties of the encaged and anesthetized single fish, especially those due to depth adaptation, were avoided by performing the measurements at nearly the precise depth of fish holding in a pen. As this was shallow, the acoustic measurements were performed ventrally, as by Röttingen[7] and Nakken and Olsen,[3] for the respective encaged and single-fish measurement types.

Performance of both encaged and single-fish measurements on successive days eliminated the need for long-term maintenance of equipment calibration. Additional performance of the calibration at least several times each day, without adjustment of equipment parameters, allowed absolute measurements to be made at all times, freeing the experiment as much as possible from unknown effects of equipment. The hydrography was also performed daily, for long-term monitoring of conditions which could change the physical condition of the fish, hence measurement results.

Finally, a large degree of redundancy was employed throughout the measurements, which were performed on two different species with a number of different echo sounding systems operating at different frequencies and different power levels. The choice of herring (*Clupea harengus* L.) and pollack (*Pollachius pollachius* (L.)) was convenient for its representation of the two classes of swimbladder-bearing fish, respectively the physostomes, which possess a duct between the swimbladder and alimentary canal, and the physoclists, which lack the same. Were depth adaptation or other behavioral modifications a problem with one species, then hopefully the very problem would be precluded by use of the other species. In any case, both kinds of acoustic measurements were performed with each species. The encaged fish measurements were performed at different times of the day, hence under different lighting conditions, over a range of densities. The single-fish measurements were conducted at similar times under similar conditions.

IV. MATERIALS

A. Experimental site

The measurements were performed from a raft anchored at the end of a sheltered fjord arm, Kvalvaagen, near Skogsvaagen on the island of Sotra, west of Bergen. The average water depth was 14 m. The typical tidal range of 0.75 m produced no measurable underwater currents anywhere near the anchorage. There were no other sources of underwater currents. The bottom was even and composed of deep, soft mud. Boat traffic in the fjord was negligible, consisting primarily in small fishing boats used only occasionally.

B. Availability of fish

The supply of living fish in good condition, undamaged by handling or even contact with the net, was ensured by the local abundance of fish and catching of these, for the experiment, by seining. Transfer of fish from the seine to holding nets or pens was accomplished by shepherding the fish over the submerged common border of the two nets when drawn together.

C. Selection of subject fish

Herring and pollack were the subjects of the measurements because of their abundance at the time of the experiment and their representation of physostomes and physoclists.

D. Measurement configuration

As noted above, two basic kinds of acoustic measurements involving fish were undertaken. These were measurements of the target strength functions of anesthetized fish and measurements of the echo energy from encaged aggregations of similar fish. Both kinds of measurements were performed with the same basic measurement and equipment configurations as those of the later "Calibration Sphere Project," reported in Ref. 24. The measurement configuration,

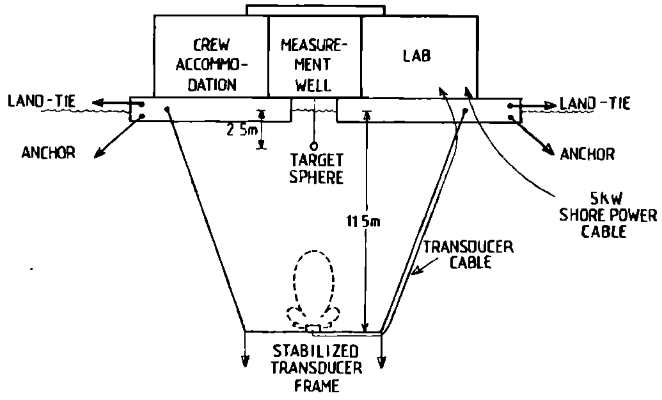

FIG. 1. Measurement configuration.

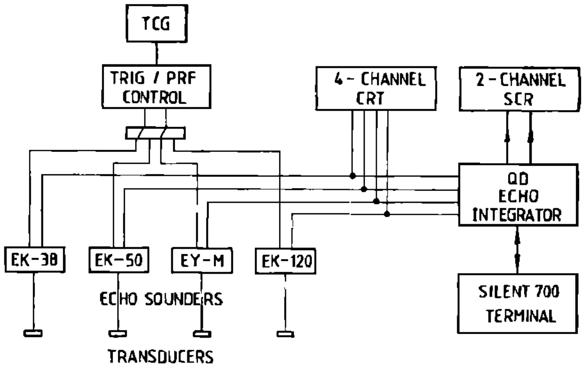

FIG. 2. Equipment configuration.

prepared to illustrate calibration of the echo sounders, is shown in Fig. 1. During fish measurements, additional equipment was present. For the single-fish target strength measurements, this was the tilting apparatus used by Nakken and Olsen,[3] although configured differently for the present investigation. The fish was held, during its tilting and measurement, at the exact 2.5 m depth and on-axis position of the calibration sphere.

During the aggregation measurements, the net cage was held on the acoustic axis with its center at the sphere position. The several net cages were designed similarly to those of Röttingen's study.[7] The height and diameter of the nearly cylindrical volume defined by the net cage were 1.10 and 0.90 m, respectively, implying a volume of 0.70 m^3.

E. Acoustic equipment

The acoustic equipment consisted primarily of four Simrad echo sounding systems and the Simrad QD digital echo integrator. Each of the four transducers had a beamwidth of approximately 20 deg at its resonant operating frequency. Some of the associated electronic equipment is indicated in Fig. 2. This is incomplete, however, for it does not include much additional, although nonessential equipment used variously during the fish measurements. This included a 14-channel instrumentation tape recorder, three hydrophones, separate transducer-signal amplifiers bypassing the receivers, a pair of four-channel oscilloscopes used for continual monitoring of signals under recording or processing, and signal amplifiers and detectors used with the hydrophones.

The parameters of the several transmit signals and power levels of the equipment are shown in Table I. Assuming an electroacoustical efficiency of 50%,[1,25] the range of acoustic power levels is seen to span the range from 17.5 to 434 W.

F. Photographic equipment

This consisted primarily of an underwater television system: a Telemation 1100 camera with specially constructed underwater housing, Cosmicar 25 mm lens, video monitor, and video recorder. During the behavioral observations, the television camera was hung at the same 2.5 m depth as the center of the net cage, but at a distance of several meters. Since the camera could not be hidden with respect to all four transducers, owing to small, but in this context, significant differences, a compromise placement was found. For this, the camera produced very weak echoes with the EK-50 and EK-120 systems, but sizeable echoes with the EK-38 and EY-M systems. In order to make clean measurements at all frequencies, the camera was generally kept in a raised position beneath the float, being lowered periodically for the crucial simultaneous acoustic and behavioral observations.

V. METHOD

Seven different series of measurements on encaged aggregations of herring or pollack were performed over a three-month period. In the first two series, fish escaped at unknown times, invalidating these and necessitating repair and reinforcement of the net cage. In the fourth and seventh series, the only two series with pollack, depth adaptation was apparently a severe problem, for the fish adopted extreme orientations approaching the frontal and the caudal. The corresponding target strength measurements on anesthetized specimens were limited to tilt angles within about 50 deg of the horizontal, hence could not be applied in a test of the linearity hypothesis. The sixth series was performed at

TABLE I. Characteristics of the four Simrad echo sounders used in the experiment.

Echo sounder	Center frequency (kHz) Nominal	Center frequency (kHz) Measured	Pulse duration (ms) Nominal	Pulse duration (ms) Measured	Peak electrical transducer power (W)
EK-38	38.0	38.0	0.6	0.64	35
EK-50	49.5	49.6	0.6	0.60	868
EY-M	70.0	68.5	0.6	0.60	89
EK-120	120.0	120.9	0.6	0.68	89

TABLE II. Numbers and biology of herring in the encaged fish measurement series and in the associated, analyzed single-fish measurements.

Type of measurement	Date	Total number of fish	Length $l \pm \Delta l$ (cm)	Weight $w \pm \Delta w$ (g)	Condition factor
Single fish target strength functions	15 July	25	27.4 ± 1.5	132.6 ± 24.0	0.006 67
Encaged aggregation	16 July	40	27.1 ± 1.5	131.2 ± 25.4	0.006 69

night, without photography, and with a high mortality, the only instance of its kind. Of the two remaining integral measurement series, only the data analysis for the third has been completed. This series, which was performed on 16 July, is the subject of the present inquiry.

Large numbers of measurements on anesthetized fish were performed before, after, and between the encaged fish measurement series. These were performed in the conventional manner, with a configuration similar to that of Nakken and Olsen,[3] but with a tauter suspension system innovated by E. Ona and A. Raknes. Because of the unknown effect of confinement on the physical state of the fish, hence on their acoustic properties, only single-fish measurements performed within one day of the subject encaged fish measurements are included in the analysis.

The number of herring involved in the two kinds of measurements associated with the encaged fish measurement series are listed in Table II together with several biological statistics. The condition factor is defined as the mean of the ratio of the weight in grams to the cube of length in centimeters for all fish in the group.

All acoustic measurements were performed absolutely, with an echo integrator that was calibrated several times daily by means of a steel ball bearing. This was later measured against copper spheres, whose target strengths are known *a priori*.[24,26]

The time-varied-gain functions of the four echo sounders were bypassed for the sake of simplicity. The nearness of the fish and the source levels of the transmitters made this amplification completely unnecessary. Fish echo levels were always high, generally exceeding the reverberation level by at least 10 dB in the mean for a single free-swimming fish.

Behavioral observations made with the underwater television were stored on videotape for later analysis. For the subject series of encaged aggregation measurements, the behavior was observed for each density, with varying degrees of resolution owing to changing lighting conditions. No artificial lighting was employed at any time during the measurements.

VI. DATA ANALYSIS AND RESULTS

The fundamental ingredients for establishing the linearity of fisheries acoustics and for verifying the echo integration method are the separate factors of Eq. (5). The derivations of these are now described.

A. Mean echo energy Av ϵ

The digital echo integrator was programmed to compute the energy in the total echo from the encaged fish aggregation. This was generally done in units of 500 pings, for which the variance was also computed. The mean and standard deviation were printed out on a typewriter/terminal at the end of each sequence of 500 pings. In the analysis of the subject encaged aggregation series, the measured average echo energy due to the empty net cage and reverberation were subtracted from the computed means. For convenience, the noise-corrected total echo energy is expressed below in units of square centimeters, to represent, in a familiar manner, the total effective scattering strength of the aggregation.

B. Number density n_0

This quantity is defined as the number of fish in the net cage. To convert this to the absolute density or number of fish per cubic meter, n_0 must be divided by the volume of the net cage; namely, 0.70 m^3.

C. Gain factor G

In the absence of time-varied gain, this is the purely geometrical factor $\exp(-2\alpha r)/r^4$, where α is the absorption coefficient at the center frequency and r is the instantaneous range of the single fish from the transducer. For the particular hydrographic conditions present during the July measurements, α was computed according to Fisher and Simmons.[27] In order of increasing frequency, $\alpha = 0.0067$, 0.0105, 0.018, and 0.035 dB/m. For convenience, G was normalized consistently with ϵ, so that the ensemble average $\langle Gb^2\sigma\rangle$ of Eq. (6) could be expressed in units of square centimeters.

D. Beam patterns b^2

The product of transmit and receive beam patterns was assumed to be given by an ideal circular transducer with total beamwidth of 20 deg at the -3-dB level. This has been found from much earlier work and from theoretical simulation to be an excellent approximation.

E. Backscattering cross section σ

The dependence of the backscattering cross section of anesthetized fish on the tilt angle was measured over a range from approximately -51 deg to $+51$ deg. Use of the logarithmic target strength TS, defined as $10 \log \sigma/4\pi$, facilitat-

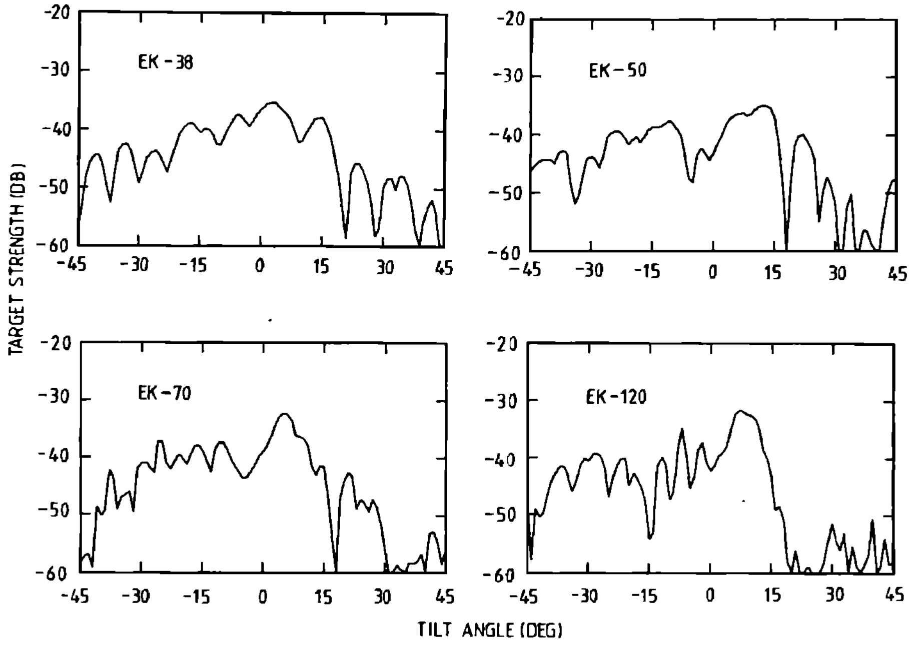

FIG. 3. Target strength functions of a 27-cm herring in ventral aspect. Positive angles denote the true head-up orientation.

ed expression of the measurements over their often large range of variation, sometimes exceeding 30 dB or a factor of 1000. These measurements are illustrated in Fig. 3 for a 27.0-cm herring measured on 15 July.

F. Fish distribution function *F*

Three dependences of this function necessary for use in Eq. (6) were obtained; namely, those of length, position, and orientation. The length distribution of the encaged fish aggregation has already been described in Table II. The spatial distribution of fish in the net cage was observed to be more or less uniform. The orientation distribution was characterized by a truncated Gaussian distribution in the tilt angle θ. Values of θ were obtained from representative still photographs extracted from the videotape. The three parameters of the distribution: the mean angle $\bar{\theta}$, standard deviation s_θ, and excursion factor n_{s_θ}, were determined by fitting a symmetrical Gaussian function to the observations. This is illustrated in Fig. 4. The results for the subject encaged aggregation series are summarized in Table III. That these are representative for the bulk of the acoustic measurements, which were made without photographic observation to avoid possible biasing by the camera echo, was confirmed by detailed examination of the acoustic records of two echo sounders for which the echo was very weak; namely, the EK-50 and EK-120 systems. For these, there was essentially no difference in the measurements with or without the camera, which is significant since the camera was favorably placed with respect to the corresponding transducer beams.

G. Ensemble average $\langle Gb^2\sigma\rangle$

Averaging of the quantity $Gb^2\sigma$ was performed according to Eq. (6) in the manner of Ref. 28. The ensemble average was computed for each anesthetized fish for which measurements of σ were available. These computations were repeated for each of the tilt angle distributions in Table III. Differences in the length distributions of the encaged aggregation and corresponding anesthetized fish were resolved by correcting the grand averages according to a quadratic length dependence of σ.

H. Results

The experimental and theoretical results are compared in Fig. 5. The confidence intervals of the experimental points are defined at the two-standard deviation level, where the standard deviation is defined as that of the series of means determined over 500-ping sequences. The variations in indi-

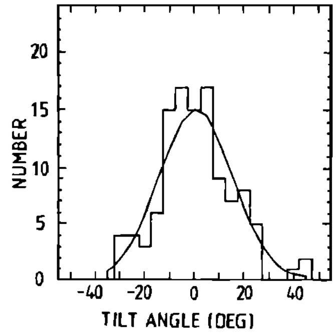

FIG. 4. Histogram of observed tilt angles for herring of number density 10, with fitted truncated Gaussian function.

TABLE III. Parameters of the tilt angle distributions of fish in the encaged aggregation measurements of 16 July.

Number density	Number of data	$\bar{\theta}$ (deg)	s_θ (deg)	n_{s_θ}
10	113	0.8	15.0	2.5
20	228	3.3	14.0	2.7
30	100	2.7	14.7	2.9
40	296	3.0	14.1	3.2

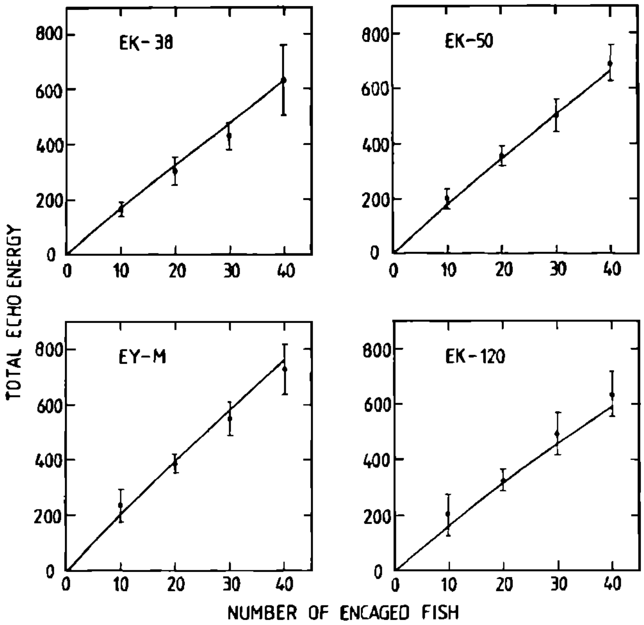

FIG. 5. Comparison of theoretical and measured values of total echo energy from the encaged herring aggregations. The echo energy has been expressed in units of square centimeters. Experimental points are indicated with confidence intervals defined at twice the estimated standard deviation. Theory is indicated by the solid line.

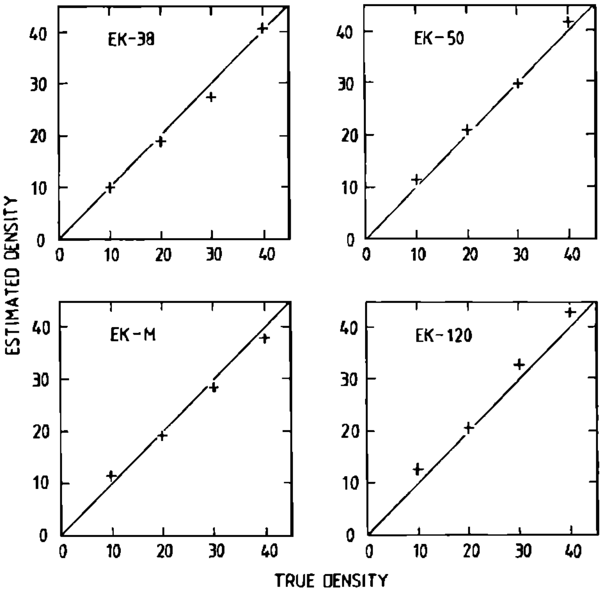

FIG. 6. Comparison of estimated and true number densities of the encaged herring aggregations. Estimates are indicated by the plus signs, and theory by the solid line.

vidual pings were much larger, of course, but as a single datum is seldom significant in acoustics, the merging of data in 500-ping sequences was considered justified.

According to Eq. (5), the number of acoustically sensed fish can be estimated by dividing the average echo energy Av ϵ by the theoretically derived ensemble average $\langle Gb^2\sigma\rangle$. This is done in Fig. 6.

VII. DISCUSSION

A. Linearity of fisheries acoustics

The linearity of fisheries acoustics is evident from the agreement shown in Fig. 5. This is confirmed by goodness-of-fit testing, with no calculated statistic being significant even at the 0.25 level. Similar results obtain if the theoretical computations are repeated for a common, density-independent behavior, which may be described by treating the data underlying Table III as though belonging to the same set.

In its simplest form, the linearity principle asserts the proportionality of total echo energy and density of fish in an aggregation.[1] This assumes, of course, that a sufficient number of observations are made under low-noise conditions. According to the present theory, this also assumes a constancy of fish behavior and the negligibility of acoustic extinction.

In general, fish behavior will vary with the density of aggregation, for, at the least, the increasing proximity of fish must change the acoustically significant orientation distribution,[28,29] if only by delimiting it. The theory remains linear, however, but in the larger sense of Eq. (5). In the absence of extinction, then, the total echo energy is the sum of independent contributions for the constituent fish of the aggregation, where the contributions depend on fish behavior and other circumstances of their observation according to Eq. (6). This finding, which is supported by the comparison of theory with experiment in Fig. 5, is embodied in the following addition theorem, which obtains under the usual conditions of ensonification by a directional echo sounder:

Theorem I: In the absence of extinction, the total echo energy from an aggregation of N fish is, in the mean of a large number of observations,

$$\epsilon_{\text{tot}} = \sum_{i=1}^{N} \epsilon_i,$$

where ϵ_i is the mean echo energy from the ith fish.

If the density, vertical extent, and mean extinction cross section of the fish are large enough so that extinction is significant, then the first theorem may be generalized by analogy with optics or quantum scattering theory. The following theorem represents a quite reasonable approximation for most applications:

Theorem II: For N fish uniformly distributed within a layer of thickness Δz, the total echo energy is, at least to the first order in the extinction parameter and in the mean of a large number of observations,

$$\epsilon_{\text{tot}} = \frac{1-\exp(-2\nu\Delta z\sigma_e)}{2\nu\Delta z\sigma_e} \sum_{i=1}^{N} \epsilon_i,$$

where ν is the fish density, σ_e is the mean extinction cross section of the fish, and ϵ_i is the mean echo energy from the ith fish, were there no extinction.

There are at least two practical applications for the second theorem, which subsumes the first; namely, in the interpretation of certain net cage measurements, cf. Refs. 7 and 30, and for correcting underestimates of density in large pelagic schools.[31] The importance of this last-mentioned instance is recognized immediately by seagoing researchers who have probably witnessed weakening, if not premature triggering, of the bottom signal by dense schools.

It is remarked in passing that the linear phenomena observed in Fig. 5 were obtained at transmitter power levels spanning the range from about 50 W to 1 kW. In regard to the nearness of target fish in the experiment and the frequent use of more powerful transmitters in acoustic surveys, the range of depths typically encountered in large-scale surveys was thus simulated.

B. Validity of target strengths derived by measurements on anesthetized fish

The fact of the agreement of theory with experiment in Fig. 5 also witnesses to another important finding. This is that the determination of fish backscattering cross sections or target strengths by measurement on anesthetized specimens is valid, at least for averaging purposes. Thus the particular methods of determining and applying target strength functions described by Midttun and Hoff,[2] Nakken and Olsen,[3] and Foote,[28] among others, are valid.

C. Verification of the echo integration method

The experiment has also verified the echo integration method of determining fish density. This is illustrated in the most direct manner in Fig. 6. While the confidence intervals have not been finally determined, these are expected to be commensurate with those of Fig. 5, or perhaps better.

Admittedly, no time-varied gain was applied to the received signals, but this was of no consequence because of the measurement geometry, chosen by design. Theoretical simulation of the results with "20 log *r*" and "40 log *r*" TVG functions confirms this. That the echo sounders otherwise performed satisfactorily was confirmed by regular calibration with a target sphere, often at intervals of several hours. A further confirmation was provided by comparing the integrated, calibrated output signals with the same echo signal intercepted at the transducer, independently amplified, and processed in the same manner as the calibrated output signal. No difference could be discerned for sufficiently strong signals. For weaker signals, the independently amplified signal was inferior, which merely reveals the difficulty of performing the function of an echo sounder without duplicating its electronics.

The fact of the fish being ensonified ventrally is similarly immaterial to the verification of the echo integration method. Because of the shallowness of the fish-holding and measurement depth, the effects of depth change and depth adaptation were negligible for the herring. The scattering nature of the fish was thereby isolated, and interpretation of the encaged fish measurements by reference to behavior and measurements on similar, anesthetized specimens, facilitated. This process was further aided by the simultaneous acoustic and photographic observations, which confirmed the constancy of behavior throughout all of the encaged fish measurements at each density and justified use of the large number of acoustic measurements made without photography and the attendant burden of integration of the camera echo, however small.

Thus, there seems little doubt that when the several factors influencing the echo from a fish aggregation are taken into account, whether intrinsic to the fish, medium, or equipment, it is possible to determine the density of that aggregation acoustically. Evidently, from Fig. 6, this determination is eminently feasible.

D. Future work

The present findings are important to research in fisheries acoustics in several ways: they confirm the basic correctness of much earlier work in principle, if not in practice, and they provide directions for future work. In particular, the effects of depth change and depth adaptation on the target strengths of fish are still unknown. Granted success with these problems, conventional measurements of the target strengths of fish presumably could be adjusted for arbitrary depths and states of adaptation. Averaging of the corresponding backscattering cross section with respect to behavior, as characterized by the spatial and orientation distributions, would provide superior numbers for immediate use in the interpretation of measurements with echo integrators.

Determination of fish behavior is thus a key link in the envisaged improved application of target strength measurements. It is hoped that fisheries biologists and behaviorists will, in the future, be able to provide quantitative descriptions of the spatial and orientation distributions of fish under surveying conditions. Failing this, acoustical schemes for the determination of behavior may be realized.[32]

VIII. CONCLUSIONS

The essential results are the following:

(1) The phenomenon of acoustic scattering by fish under surveying conditions is strictly linear.

(2) Mean acoustic backscattering cross sections of living, free-swimming fish can be determined from measurements on representative anesthetized specimens.

(3) The echo integration method of determining fish density is valid.

A natural sequel to the present study would be elucidation of depth-induced effects. Such knowledge, when added to the present store and guided by descriptions of fish behavior, should effect an immediate, significant improvement in the acoustic estimation of fish abundance.

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