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ASSESSING THE EFFECT OF INTRA-HAUL CORRELATION AND VARIABLE DENSITY ON POPULATION ESTIMATES FROM MARINE SURVEYS

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ABSTRACT

In a previous paper (Pennington and Vølstad, Biometrics 47, 1991) it was suggested that reducing the size of the sampling unit generally used in marine surveys could increase the precision of abundance estimates. But if unit size is reduced, fewer animals would be caught during a survey. Concern has been expressed that this reduction in total catch would lower the precision of estimates of population characteristics, such as mean fish length, of importance for stock management. In this paper we examine the effect of sampling unit size, intra-cluster correlation and variable density on the precision of population estimates. Based on an examination of some survey data, it appears that reducing the size of the sampling unit generally employed and using the time saved to take samples at more locations could also yield more precise population estimates.

Key words: Marine surveys; Intra-cluster correlation; Ratio estimator; Jackknife; Sampling unit; Survey design.

1. Introduction

Marine trawl surveys are routinely used to measure the abundance or relative abundance of many fish stocks and for estimating population characteristics such as mean length and age. This information forms the basis for managing many fisheries throughout the world. For most surveys a standard trawl is towed for usually a half hour or longer at each selected station (see, e.g., Sparre, Ursin and Venem, 1989). Previous results (Pennington and Vølstad, 1991) indicate that reducing tow duration, i.e. the size of the sampling unit commonly used, and appropriately increasing the number of locations sampled could result in more precise abundance estimates. But this also reduces a survey's total towing time and hence the number of fish caught. For example, 100 ten-minute tows or 77 thirty-minute tows can be made during a routine survey on Georges Bank. The former strategy will produce more precise abundance estimates, but on average more than twice as many fish will be caught with the latter.

Concern has been expressed that if the size of the sampling unit is reduced, too few fish will be caught, especially when abundance is low, to provide adequate estimates of population parameters. But the perception of what is a sufficient sample size is usually based on the number of fish caught, which are often assumed to be a random sample from the population, and no account is taken of the effect of intra-haul correlation. It is well known that even low levels of intra-cluster correlation can greatly increase the variance of an estimate as compared with that from simple random sampling (see, e.g., Hansen, Hurwitz and Madow, 1953).

In this paper the effect of reducing the size of a survey's sampling unit on the precision of an estimate of the mean value of some quantity, such as length, age or weight of stomach contents per individual is examined. Motivated by experimental results, the variance of an estimate is related to unit size in section 2, and then the effect of reducing tow duration to that appropriate for density estimates is assessed.

As an example, the precision of survey estimates of the mean length of Georges Bank haddock is examined in section 3. The most striking feature of these data is that even though a total of several thousand fish from 60 or more locations were often measured, the same precision could have been obtained if it were possible to randomly sample as few as 30 fish from the population. This imprecision is caused by large intra-haul correlation made worse by the fact that the density of the stock varies greatly from one location to another. Reducing the unit size for these surveys would not only increase precision but also reduce the number of fish that need to be measured.

The analysis also provides further confirmation that the usual approximate formula for the standard error of the ratio estimator (see, e.g., Cochran, 1977, p. 32) can appreciably underestimate the true value (Rao, 1968; Wu and Deng, 1983). In contrast, the jackknife estimate of the standard error, as suggested by Wu and Deng (1983), appears to produce more dependable estimates.

It is concluded in section 4 that even if tow duration for these surveys is reduced the resulting estimates will not be particularly precise. This is because the sampling trawl used in standard surveys is basically the one used by fishermen. Commercial equipment is designed to catch as many fish as possible at one spot. But for assessment purposes, due to the nature of fish distributions, it appears that the best strategy is to sample a few fish from as many locations as feasible.

2. The Effect of Unit Size on Precision

Suppose n stations are chosen randomly in an area and at each station a trawl is towed for a fixed amount of time. Let m_i denote the number of fish caught at the i^{th} station (m_i can equal 0). Then if x_{ij} is some measurement on each individual, the mean of x may be estimated using the usual ratio estimator,

$$\bar{x}_r = \Sigma\Sigma x_{ij} \,/\, \Sigma m_i.$$

We first express the variance of $\bar{x}_r$ for a fixed unit size in a form in which the sources of its variability can be assessed. Then we analyze the effect on $\mathrm{Var}(\bar{x}_r)$ of changing a standard survey's unit size to one that is efficient for estimating density.

The variance of $\bar{x}_r$ may be written as the sum of two components or

$$V(\bar{x}_r) = E_{\mathbf{m}}\{V(\bar{x}_r|\mathbf{m})\} + V_{\mathbf{m}}\{E(\bar{x}_r|\mathbf{m})\}, \tag{2.1}$$

where **m** denotes the vector of catches (see, e.g., Rao, 1973, p. 97).

For the first component, it can be shown that

$$V(\bar{x}_r|\mathbf{m}) = \sigma_x^2\{1+(\bar{m}-1+s_m^2/\bar{m})\rho\}/\bar{m}n,$$

where σ_x^2 is the population variance of x, ρ is the intra-haul correlation coefficient, and $\bar{m}$, s_m^2 are the sample average and variance of the m_i's, respectively. For large n its expectation is approximately equal to

$$\sigma_x^2\{1+(\overline{M}-1+\sigma_m^2/\overline{M})\rho\}/\overline{M}n, \tag{2.2}$$

where $\overline{M} = E(\bar{m})$ and $\sigma_m^2=E(s_m^2)$.

The second component in (2.1) is the result of any correlation between cluster size and $\bar{x}_r$. For large n, $E(\bar{x}_r|\mathbf{m})$ will be approximately equal to $\mu_x+\alpha(\bar{m}-\overline{M})$, where $\mu_x= E(x)$, and α is a constant which will equal zero if $\bar{x}_r$ and $\bar{m}$ are uncorrelated. Therefore

$$V\{E(\bar{x}_r|\mathbf{m})\} \doteq \alpha^2\sigma_m^2/n. \tag{2.3}$$

Thus $Var(\bar{x}_r)$ is approximately equal to

$$\sigma_x^2\{1+(\overline{M}-1+\sigma_m^2/\overline{M})\rho\}/\overline{M}n + \alpha^2\sigma_m^2/n \tag{2.4}$$

and so is a function of σ_x^2, σ_m^2, n, $\overline{M}$, ρ and α.

Based on several trawling experiments, it was found that to an adequate approximation (see Pennington and Vølstad, 1991, for details)

$$\sigma_m^2 = m_0 t + b(m_0 t)^2,$$

where $\overline{M} = m_0 t$ is the mean catch per tow of duration t and b is a constant greater than zero. It was also shown that for a survey of fixed duration, C, the number of stations, n_t, which can be sampled with tow duration t is approximately defined by

$$C = (c_1+t)n_t + c_2\sqrt{n_t}\ , \qquad (2.5)$$

or

$$n_t = [\{(c_2^2 + 4(c_1+t)C)^{1/2} - c_2\}/2(c_1+t)]^2, \qquad (2.6)$$

where c_1 is time needed to set and retrieve the trawl at each station and c_2 is a constant which depends on the area of the survey region. Finally, that the optimum length of tow, t_0, for density estimation (i.e. the one that minimizes $\sigma_m/\overline{M}\sqrt{n_t}$) is the iterative solution of (2.6) and

$$t = \{(c_1 + c_2/2\sqrt{n_t})/m_0 b\}^{1/2}. \qquad (2.7)$$

We here assume that at a station fish are fairly well mixed and hence that ρ and α do not change with tow duration. This is supported by some experimental results. For example, estimates of the intra-haul correlation for length measurements do not appear to vary significantly with t (Godø, Pennington and Vølstad, 1990).

Since α is assumed constant and n_t decreases as t increases, (2.3) is an increasing function of t. The tow duration, t_0', which minimizes (2.2) subject to the constraint (2.5) is given iteratively by (2.6) and

$$t = \{(c_1 + c_2/2\sqrt{n_t})/m_0(b+1)\rho\}^{1/2}. \tag{2.8}$$

For $t > t_0'$, $\mathrm{Var}(\bar{x}_r)$ is an increasing function of t. From (2.7) and (2.8) it can be seen that $t_0' = \{b/(1+b)\rho\}^{1/2} t_0$. If $\alpha = 0$, then t_0' minimizes $\mathrm{Var}(\bar{x}_r)$. If the variance component (2.3) is relatively large, which does not appear to be the case for the marine surveys we have examined, then the tow duration which minimizes (2.4) given the constraint (2.5) can be found numerically and compared with t_0.

In practice, the real problem is not to find the exact tow duration that minimizes a particular quantity, but to decide whether, for example, a ten-minute tow will generally be more efficient than a thirty-minute tow. This is not only because a marine survey has many objectives, but also because the optimum tow duration is a function of population parameters and available resources that change over time. Fortunately the values of t_0 and t_0' vary as the square root of the parameters and the resulting variance curve is fairly flat around its minimum.

3. An Example: Determining Tow Duration for a Survey on Georges Bank

We show in this section how historical survey data can be used to assess the appropriate unit size for future surveys. Estimates of the mean length of Georges Bank haddock are only considered here, but in practice all variables of interest can be treated in a similar fashion and a compromise unit size selected.

Fall trawl surveys have been conducted on Georges Bank, a region off the northeast coast of the U.S.A., by the National Marine Fisheries Service since 1963. The bank is divided into areal strata and within each stratum a number of stations, approximately proportional to stratum area, are randomly selected. A cruise track is then determined which minimizes the total travel time between stations on the entire bank and at each station a trawl is towed for thirty minutes. The surveys usually take six to seven days to complete.

In section 3.1 the precision of estimates of the mean length of haddock obtained by the current survey design is examined. We assume that the sample of stations is approximately a random one from the entire area. Sampling is done proportional to stratum area because the spatial distribution of fish changes dramatically from year to year. Therefore in practice it is necessary to choose a unit size which will be adequate for the entire bank rather than for particular subareas.

We discuss in section 3.2 the effects of areal stratification on these estimates of mean length. In section 3.3 we determine a tow duration that appears to be more suitable for estimating mean length and density than the present standard of thirty minutes.

3.1. Precision Obtained with the Current Tow Duration

In Table 1 are ratio estimates of the mean length of haddock on Georges Bank for 1963 to 1988. Estimates of their standard errors were made using the usual approximation and the jackknife estimator (Cochran,1977, p. 32 and p. 179, respectively). The approximation was on average 18% smaller than the jackknife values (Table 1).

It has been suggested that the usual approximation can seriously underestimate the true standard error (see, e.g., Rao, 1968; Cochran, 1977; or Effron, 1982) and that the jackknife estimator is generally preferable (Wu and Deng, 1983).

To check if the jackknife estimates for these data fairly reflect the true level of precision, we ran several simulations based on the observed data as in Wu and Deng (1983). Since the effective sample size is determined by the number of positive catches, years with the largest number of such tows were used in the simulations. For each year selected, 2000 samples of size 30 were randomly chosen from the positive values. The results are in Table 2.

As Wu and Deng (1983) observed, the jackknife estimator appears to provide consistently more accurate estimates of the standard error and nominal 95% confidence intervals. But for samples of size 30, which is near the effective sample size for many of the years (Table 1, col. 3), the jackknife estimate may also overstate the precision obtained.

In the last two columns of Table 1 we compare the actual number of fish measured with the number that would have been needed to obtain the same precision if fish could be randomly sampled. This was done using the jackknife estimate of the standard error and the usual estimate of the population standard deviation for length (Table 3, col. 2). Though these are rough estimates, they indicate that if fish could be sampled randomly, many fewer would be needed. In fact, the number appears often to be less than the number of tows that caught haddock (Table 1, col. 3). The imprecision of the estimates of the mean length is due to high intra-haul correlation and large between tow variability in catches (Table 3) which greatly inflates the variance as compared with random sampling (equation 2.4). It is not only the mean that is imprecisely estimated, of course, but the entire length distribution of the population.

3.2. Effects of Stratification

To take into account the areal stratification of trawl stations, a combined ratio estimator (Cochran, 1977, p.165) would be appropriate.

Or

$$\bar{x}_{st} = \Sigma w_k \bar{y}_k \,/\, \Sigma w_k \bar{m}_k \ ,$$

where for the k^{th} stratum: w_k is the proportion of survey area in the stratum, $\bar{y}_k$ is the average total fish length per tow, and $\bar{m}_k$ is the average catch per tow. Though seemingly awkward, this type of estimator is necessary because the proportion of fish in each stratum is unknown.

The average value of the jackknife estimates of the standard error of $\bar{x}_{st}$ for the haddock data was 3.52 as compared with 3.54 obtained assuming a simple random sample of stations. As would be expected, the average value of ρ within a stratum was smaller (.33) than the estimates for the entire area (.68).

The reason that this decrease in ρ did not result in more precise estimates can be seen from equation (2.1). For suppose the strata were chosen small enough so that in each stratum $\rho = 0$. Then $E_m\{V(\bar{x}_{st}|\mathbf{m})\}$ could be relatively small, but $V_m\{E(\bar{x}_{st}|\mathbf{m})\}$ would increase since differences in mean length among the strata now become a factor.

3.3. Selecting an Appropriate Tow Duration

The sampling trawl used for the Georges Bank surveys takes 30 min to set and retrieve or c_1= 30. The areal parameter, c_2, is 530 min. In Table 3, col. 6 are estimates of m_0b for each year. Using equations (2.5), (2.6) and (2.7) it was found that the optimum tow duration for density estimates is less than 7 min for all years except for 1969 (17 min) and 1971 and 74 (10 min). For most years the optimum was less than 5 min (20 of 26).

For the length data there is relatively little correlation between the average length of fish in a cluster and cluster size, and thus α is effectively zero for all years. Therefore $\mathrm{Var}(\bar{x}_r)$ is minimized if t is $\{b/(b+1)\rho\}^{1/2}$ times the optimum tow duration for density. Estimates of this factor are in Table 3, col. 7. To check if equation (2.4) is useful for designing future surveys, estimates of the population parameters for the 30-min tows were substituted into equation (2.4, $\hat{\alpha} = 0$) [Table 1, col. 7].

Based on the above it appears that the current 30-min tow duration could be safely reduced to 10 min. Tows less than 10 min are not considered feasible (or acceptable) at this time because for very short tows the sampling properties of the standard trawl are not known. To measure the possible gains to be had by using 10-min tows, estimates from equation (2.4) of $\mathrm{Var}(\bar{x}_r)$ for 10-min tows divided by that for 30-min are given in Table 3, col. 10 as are ratios of $\sigma_m^2/\bar{M}^2 n_t$ for the density estimates, col. 9. In col. 8 are estimates of n_{10} for each year.

4. Conclusions

Reducing tow duration for marine surveys should result in more precise estimates of population parameters and of abundance. However given the high cost of these surveys, the standard errors would still be relatively large and the effective sample size for population estimates would be small compared with the number of fish sampled. The problem is that apparently fish should be collected from as many locations as possible, but the sampling gear, which is essentially the one used by fishermen, is designed to maximize catch at one location. The gear is fairly large and is towed by fishermen for two hours or longer. Consequently, it is not primarily designed to be rapidly set and retrieved.

But this limits the number of stations that can be sampled during a survey. The variance of the estimates was approximately reduced by a factor of n_{30}/n_{10} if 10-min rather than 30-min tows were used. Further gains could be had if the time to set and retrieve the net, or c_1, were decreased. For example if $c_1 = 5$, then 165 stations could be sampled on Georges Bank using 10-min tows versus 77 for the present design. A smaller value of c_1 would also significantly reduce the optimum tow durations (eqs. 2.7, 2.8).

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TABLE 1

Summary statistics for estimating the mean length of haddock on George Bank. The last two columns contain the number of fish actually measured and the estimated number needed to obtain the same precision if fish could be randomly sampled. The standard errors of $\bar{x}_r$ were calculated using the usual approximation, jackknifing (Cochran, 1977, p. 32 and p. 179) and by substituting parameter estimates into equation (2.4).

Year	n	Num. of non-zero tows	$\bar{x}_r$	Estimated S.E. Approx.	Jack	Eq. (2.4)	Total num. of fish	Random sample
63	73	62	25.3	2.4	2.6	3.2	7083	38
64	73	60	33.7	1.1	1.1	1.4	8411	83
65	76	67	38.9	0.6	0.6	1.0	4725	152
66	74	53	40.0	2.8	3.1	2.4	1505	20
67	78	59	49.2	2.8	3.4	2.4	893	10
68	80	36	57.0	1.0	1.0	1.9	414	97
69	84	36	52.8	3.2	3.4	3.9	157	29
70	81	40	50.7	3.1	4.8	4.7	450	9
71	84	40	34.8	6.4	7.3	6.1	279	13
72	85	49	28.6	3.5	4.0	4.5	639	24
73	84	31	34.8	2.5	2.7	4.0	796	33
74	85	32	38.8	3.2	3.6	4.0	247	21
75	84	58	24.6	4.7	5.3	4.6	1955	12
76	78	36	34.6	0.8	1.0	2.8	3727	56
77	112	56	45.2	0.7	1.2	2.1	4688	28
78	175	124	33.1	4.2	4.7	4.3	4353	16
79	171	100	35.4	0.5	1.3	3.8	12208	28
80	102	62	29.3	5.0	6.5	5.1	3927	7
81	82	43	43.9	1.9	2.1	2.2	930	33
82	79	40	45.8	4.3	4.8	4.7	381	16
83	81	52	32.5	3.4	3.7	4.2	772	25
84	80	30	37.0	2.0	2.9	3.7	576	12
85	77	41	25.6	2.3	2.9	3.9	1136	21
86	79	22	39.9	2.8	3.6	3.8	679	9
87	77	25	31.2	7.1	10.7	7.3	419	3
88	77	25	43.1	3.3	3.8	3.5	592	12
			Avg.	2.92	3.54	3.62		

TABLE 2

Simulation results for assessing the performance of the usual approximation and the jackknife estimator of the standard error of the ratio estimator. For each year selected, 2000 samples of size 30 were generated from the positive catches.

Year	$CV_{m\neq0}$	True $\sqrt{MSE}$	Avg. S.E. Approx.	Avg. S.E. Jack	Percent deviation from true $\sqrt{MSE}$ Approx.	Percent deviation from true $\sqrt{MSE}$ Jack	Nominal coverage 95% confidence interval Approx.	Nominal coverage 95% confidence interval Jack
63	1.61	3.63	3.15	3.66	−13	1	89.2	91.8
64	1.41	1.47	1.32	1.44	−10	−2	89.1	90.5
65	1.42	0.83	0.78	0.84	−6	1	90.7	91.8
75	1.88	6.11	4.56	5.52	−25	−10	70.8	73.9
78	2.52	6.92	5.26	7.04	−24	2	78.1	87.4
79	6.81	2.84	1.64	2.53	−42	−11	81.8	89.7
80	2.48	7.14	4.59	6.52	−36	−9	70.5	75.2

TABLE 3

Parameter estimates for determining the effect of reducing unit size for the George Bank surveys. In column 9 are estimates of the resulting reduction in $(cv_m)^2/n_t$ for density, R_1, and in the last column that for $Var(\bar{x}_r)$, R_2.

Year	$\hat{\sigma}_x$	$\hat{\rho}$	$\bar{m}$	s_m	$\widehat{m_0 b}$	$\sqrt{\hat{b}/(\hat{b}+1)\hat{\rho}}$	n_{10}	R_1	R_2
63	16.1	.68	97.1	173	10.4	1.1	94	.78	.78
64	9.7	.41	115.2	187	10.1	1.3	94	.78	.74
65	7.4	.40	62.2	97	5.0	1.3	99	.78	.76
66	13.6	.58	20.3	34	1.9	1.1	95	.80	.81
67	10.6	.68	11.5	26	1.9	1.1	101	.80	.81
68	10.1	.36	5.2	13	1.1	1.6	104	.82	.90
69	17.9	.83	1.9	4	.2	1.0	109	1.00	.98
70	14.1	.56	5.6	21	2.7	1.3	105	.79	.80
71	25.5	.79	3.3	7	.5	1.0	109	.86	.86
72	19.6	.77	7.5	4	1.2	1.0	110	.81	.82
73	15.2	.55	9.5	29	2.9	1.3	109	.79	.79
74	16.1	.76	2.9	7	.5	1.1	110	.86	.88
75	17.5	.90	23.3	55	4.3	1.0	109	.75	.78
76	7.3	.64	47.8	194	26.3	1.2	101	.77	.78
77	6.3	.48	41.9	216	37.2	1.5	148	.76	.81
78	18.5	.93	24.9	76	7.7	1.0	235	.75	.75
79	7.0	.62	71.4	638	189.7	1.3	229	.75	.75
80	16.0	.89	38.5	126	13.7	1.0	134	.77	.77
81	11.7	.54	11.3	23	1.5	1.2	106	.80	.83
82	18.5	.71	4.8	12	.9	1.1	102	.83	.84
83	18.6	.78	9.5	20	1.3	1.0	105	.81	.83
84	9.9	.65	7.2	28	3.7	1.2	104	.79	.79
85	13.4	.85	14.8	38	3.3	1.0	100	.78	.79
86	10.2	.73	8.6	32	4.0	1.1	102	.79	.79
87	18.1	.90	5.4	19	2.3	1.0	100	.80	.79
88	12.5	.80	7.7	20	1.7	1.0	100	.80	.80