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ACOUSTIC SAMPLING VOLUME FOR COD

by

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ABSTRACT

This is expressed through the effective equivalent beam angle ψ for cod as represented by measurements of the tilt angle dependence of target strength. The functional form of ψ is approximated for each of several behaviour modes.

RESUME: VOLUME D'ECHANTILLONNAGE POUR LA MORUE

Il est exprimé par l'angle équivalent effectif du faisceau ψ pour la morue, comme représenté par les mesures de dépendance de l'index de réflexion des angles d'inclination. La forme fonctionnelle de ψ est calculée approximativement pour chacun des plusieurs modes de comportement.

INTRODUCTION

At this meeting in 1988 an explicit expression was given for the effective equivalent beam angle (Foote 1988). This related the angle ψ to the product transmit-and-receive beam patterns b^2, a gain or geometric factor g, backscattering cross section σ, and threshold t through the formula

$$\psi = \iint b^2 H(gb^2\sigma - t)\ dF\ d\Omega \quad , \tag{1}$$

where $H(\cdot)$ is the Heaviside step function, equal to 0, ½, or 1, as the argument is less than, equal to, or greater than zero, respectively. The integration is performed with respect to the distribution F of scatterer position and orientation in the beam over a spherical surface centered at the transducer. It was found that when the threshold vanishes or is negligible with respect to the echo strength $gb^2\sigma$, ψ equals or approaches its nominal value

$$\psi_o = \int b^2 \, d\Omega \quad . \tag{2}$$

When the echo strength of a scatterer located on the acoustic axis and in its most favorable orientation just equals the threshold, then ψ vanishes. The transition from ψ_o to 0, from near range to the maximum detection range, is monotonic and smooth.

Computations were presented in the cited paper for both ideal point scatterers and a number of individual specimens of cod (<u>Gadus morhua</u>), as represented by the respective measured target strength functions of tilt angle (Nakken and Olsen 1977). These showed precisely how ψ is expected to vary with range for the several scatterer types.

This work is a continuation of the first, but with the aim of developing simple formulae, in lieu of tabulations, for use in estimating cod stock size from acoustic survey data. A general method for approximating a function that decreases smoothly and monotonically from its maximum value to zero over a finite domain is presented. Its application to cod is then described, and results are given, for the range dependence of ψ, through equations distinguished by length class and behaviour mode. Limitations are mentioned, and the nature of ψ is further discussed.

METHOD

The present object is approximation of the range dependence of the effective equivalent beam angle $\psi(r)$. For simplicity, ψ is normalized to its limiting value for vanishing threshold, ψ_o, and the ratio designated $\hat{\psi}=\psi(r)/\psi_o$.

As already mentioned, $\hat{\psi}$ decreases smoothly and monotonically, from 1 to 0 as the range r increases from 0 to r_m, the maximum detection range for the particular scatterer and threshold value. This mathematical ogee resembles a low-pass filter characteristic, hence

$$\hat{\psi}_1 = 1 - [1+(q\Delta)^2]^{-1} \quad , \tag{3}$$

is a first approximation to $\hat{\psi}$, where $\Delta=r/r_m-r_m/r$, and q is a measure of the steepness of falloff of the function.

Since $\hat{\psi}_1$ contains only one free or adjustable parameter, its representation of $\hat{\psi}$ will in general be quite approximate. To improve on this, a Fourier series is fitted to the residual function

$$\hat{\psi}_{res} = \hat{\psi} - \hat{\psi}_1 \quad , \tag{4}$$

namely the cosine series, truncated at finite j=n,

$$\hat{\psi}_2 = a_o + \sum_{j=1}^{n} a_j \cos(j\pi r/r_m) \quad , \qquad (5)$$

where

$$a_o = \frac{1}{r_m} \int_0^{r_m} \hat{\psi}_{res}(r)\, dr \quad , \qquad (5a)$$

and, for all $j \neq 0$,

$$a_j = \frac{\int_0^{r_m} \hat{\psi}_{res}(r) \cos(j\pi r/r_m)\, dr}{\int_0^{r_m} \cos^2(j\pi r/r_m)\, dr} \quad . \qquad (5b)$$

This last expression for a_j is not reduced, to indicate how the evaluation should proceed numerically when $\hat{\psi}_{res}$ is represented not by a continuous function, but by a finite set of numbers.

The effective equivalent beam angle $\hat{\psi}(r)$ is thus approximated as follows:

$$\psi(r)/\psi_o \approx \hat{\psi}_1(r) + \hat{\psi}_2(r) \quad . \qquad (6)$$

Quantities to be determined are the factor q in $\hat{\psi}_1(r)$ and number of terms n in $\hat{\psi}_2(r)$.

Two approaches to the determination of q are outlined. Both are designed to get simply at this. Firstly, the quantity is determined by fitting $\hat{\psi}_1(r)$ to $\hat{\psi}(r)$ at the inflection point $r=r_i$. Solving equation (3),

$$q = q_i = \frac{1}{|\Delta_i|} \left| \frac{\hat{\psi}(r_i)}{1-\hat{\psi}(r_i)} \right|^{1/2} \quad , \qquad (7a)$$

where $\Delta_i = r_i/r_m - r_m/r_i$. Secondly, the quantity is determined at the 50%-point $r=r_{50}$, where $\hat{\psi}_1(r)=\hat{\psi}(r)=0.5$, thus with

$$q = q_{50} = \frac{1}{|\Delta_{50}|} \quad , \qquad (7b)$$

where $\Delta_{50} = r_{50}/r_m - r_m/r_{50}$.

Several considerations are important in deciding where to end the cosine series in equation (5). A good fit to $\hat{\psi}_{res}$ is desired, but not at the expense of having a large number of terms. To maximize the first and minimize the number of terms, n is chosen with regard to certain objective criteria. It is clear from the nature of $\hat{\psi}$ that small values are rather more uncertain than large values. This is particularly evident from computations of $\hat{\psi}_{res}$, which oscillates, more or less, with increasing amplitude as r increases. It is thus reasonable to restrict the range of fitting to values of $\hat{\psi}$ exceeding, say, 0.2. The number of terms can now be determined by requiring that the fit be sufficient to incur a relative error not exceeding, say, 5%.

COMPUTATIONS

Several computational parameters have the same values as previously held (Foote 1988). The medium is defined by the sound speed, 1470 m/s, and absorption coefficient α=0.0106 dB/m at the transducer frequency of 38 kHz. For the assumed case of single-fish detection, the gain factor g in equation (1) is simply $10^{-\alpha r/5} r^{-4}$. The maximum detection range r_m is assumed to be 400 m. The beam pattern is due to an ideal circular piston of full beamwidth 8 deg between opposite -3-dB levels. Backscattering cross sections are derived from measurements of the tilt angle dependence of target strength by Nakken and Olsen (1977). The data for cod, in particular, are used. As tabulated by Foote and Nakken (1978), 68 specimens spanning the length range 6.7-96 cm are represented. The fish behaviour is characterized by a uniform spatial distribution and an orientation distribution whose principal part is that of the tilt angle. Three normal distributions of tilt angle are used: N(-4.4,16.2) deg, as observed for cod *in situ* (Olsen 1971); N(0,5) deg, as observed for penned saithe (Foote and Ona 1987); and N(0,10) deg, meant to be intermediate to the others.

For each tilt angle distribution, $\hat{\psi}(r)$ is computed in the previous manner for each fish over the range [0,400] m at 4-m intervals. The results are averaged for each of the following four length groups: [5,15), [15,35), [35,55), and [55,100) cm, with statistics shown in Table 1. The averaged and normalized functions $\hat{\psi}(r)$, which are distinguished by length group and behaviour mode, serve as the bases for the analyses indicated by equations (3)-(5).

Table 1. Statistics of the four length groups. All length measures are in units of centimeters. The minimum and maximum lengths in a group are denoted ℓ_{min} and ℓ_{max}. The sample size is denoted n_s. The standard deviation is $\Delta\ell$.

Nominal length range	ℓ_{min}	ℓ_{max}	n_s	$\bar{\ell}$	$\Delta\ell$	$\overline{\ell^2}^{\frac{1}{2}}$
5 - 15	6.7	11.5	11	8.6	1.5	8.7
15 - 35	20.5	34.0	18	29.2	3.8	29.5
35 - 55	35.0	52.3	20	42.2	5.5	42.6
55 - 100	55.0	96.0	19	67.0	10.1	67.8

RESULTS

These are presented in Table 2. Included are the relative range r_5/r_m, for fitting with error less than 5%, and the relative value $\hat{\psi}$ at r_5.

Repetition of the computations for a full beamwidth of 5 deg at the -3-dB level gives very similar results. These may, in fact, be regarded as identical to the results presented for the 8-deg beamwidth, at least to the likely level of precision.

Absolute values of $\psi(r)$ may be derived from the corresponding relative values $\hat{\psi}(r)$ by multiplying by ψ_o. For the 8-deg beamwidth, this is 0.0108 sr.

DISCUSSION

The criteria described in the Method section are mostly fulfilled by the sum of the low-pass-type function and seven-term Fourier cosine series in equations (3) and (5), respectively. The results could be improved by adding more terms to the cosine series. For an eleven-term series, with n=10, $\hat{\psi}(r)$ can be fitted to within 5% error for $\hat{\psi}(r)>0.13$ in the worst case and 0.01 in the best case.

Both the inflection point and 50%-point have been used in computing $\hat{\psi}_1$ in equation (3). The difference is very slight, with perhaps one less cosine term required with use of the inflection point.

Several trends are apparent in the results in Table 2. (1) For a given behaviour mode, q_i generally decreases with increasing length group. This reflects the less directional pattern of scattering by small fish compared to that from large fish. The angle $\psi(r)$ thus remains near the nominal transducer value until the maximum detection range is closely approached, when the function falls rapidly to its negligible threshold value. For larger fish the scattering pattern is quite directional, and the transition from nominal to zero values is more gradual, hence with diminished value of q_i. (2) Similarly, with increasing tilt angle range, the transition is more gradual, and q_i decreases for fish of the same length group.

Inspection of the coefficients of the Fourier cosine series discloses no apparent trend. This is perhaps not to be expected either, for the residual function in equation (4) shares and actually accentuates irregularities in $\hat{\psi}(r)$, which is moreover affected by the smallness of the sample sizes indicated in Table 1.

The nature of ψ seems to be understood. As with target strength, the importance of fish behaviour is unmistakable. The tilt angle distribution in particular is implicated (Foote 1980). Just as tilt angle can be inferred from measurements of target strength (Foote and Traynor 1988), so might it be inferred from measurements of the effective equivalent beam angle. Observational material is wanted.

Table 2. Parameters of fitted functions in equations (3) and (5), namely q_i and a_j, $j=0,1,\ldots,6$, for each of three tilt angle distributions, characterized by the mean $\bar{\theta}$ and standard deviation s_θ in degrees, and each of four length groups. Additional tabulated quantities are the relative ranges r_i/r_m for the inflection point and r_5/r_m for the first point at which the fitted function deviates from $\hat{\psi}$ by 5%, and value of $\hat{\psi}$ at r_5. The limits of applicability of the approximation in equation (6) are thus defined: $r<r_5$ and $\hat{\psi}(r)>\hat{\psi}(r_5)$.

					Fourier cosine series coefficients								
$\bar{\theta}$	s_θ	ℓ(cm)	r_i/r_m	q_i	a_0	a_1	a_2	a_3	a_4	a_5	a_6	r_5/r_m	$\hat{\psi}(r_5)$
-4.4	16.2	9 ± 2	0.89	1.98	-0.061	0.018	0.061	-0.027	0.014	0.002	-0.003	0.88	0.20
-4.4	16.2	29 ± 4	0.53	0.88	0.012	-0.001	0.005	-0.009	-0.006	0.006	-0.006	0.86	0.10
-4.4	16.2	42 ± 6	0.53	0.82	0.017	0.009	0.001	-0.017	-0.012	0.005	-0.003	0.87	0.07
-4.4	16.2	67 ± 10	0.51	0.87	0.013	0.023	-0.001	-0.032	-0.008	0.006	0.000	0.88	0.06
0	10	9 ± 2	0.92	2.75	-0.018	0.023	0.005	-0.020	0.016	-0.009	0.004	0.97	0.04
0	10	29 ± 4	0.65	1.12	0.015	-0.012	0.010	-0.007	-0.010	0.009	-0.008	0.85	0.17
0	10	42 ± 6	0.63	1.05	0.015	-0.001	0.001	-0.011	-0.008	0.006	-0.004	0.87	0.11
0	10	67 ± 10	0.66	1.08	0.020	0.011	-0.015	-0.020	0.002	0.004	-0.002	0.88	0.08
0	5	9 ± 2	0.92	3.56	-0.008	0.017	-0.006	-0.012	0.015	-0.012	0.007	0.93	0.21
0	5	29 ± 4	0.77	1.41	-0.008	-0.006	0.032	-0.015	-0.005	0.007	-0.008	0.85	0.22
0	5	42 ± 6	0.72	1.24	0.014	-0.002	-0.005	-0.010	0.001	0.006	-0.005	0.87	0.14
0	5	67 ± 10	0.75	1.17	0.022	0.009	-0.023	-0.014	0.004	0.000	0.001	0.89	0.08

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