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WIDEBAND BEAM PATTERN

by

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ABSTRACT

Transducer directivity is important in the basic acoustic surveying methods of echo counting and integration. In quantifying the directivity, the beam pattern is often computed assuming ideal single-frequency operation. The validity of this for typical pulsed signals and receivers is examined. A wideband beam pattern is defined and computed for several different combinations of transducer geometry, transmit pulse duration, and receiver frequency response function. For each, the directivity index and equivalent beam angle are computed and compared with the respective single-frequency estimates.

RESUME: DIAGRAMME D'EMISSION POUR BANDE LARGE

La directivité d'un transducteur est importante dans les méthodes acoustiques de compte d'échos et d'intégration d'échos. En quantifiant la directivité, le diagramme d'émission est souvent calculé en supposant une seule fréquence. La validité de cette supposition est examinée pour des signaux émis et pour des antennes de réception typiques. Un diagramme d'émission pour bande large est défini et calculé pour plusieurs combinaisons de géométrie des transducteurs élémentaires, durée des pulsations émises et fonction de réponse de fréquence de l'antenne de réception. Dans chaque cas, l'index de directivité et l'angle équivalent du faisceau sont calculés et comparés avec les estimations respectives pour une seule fréquence.

INTRODUCTION

An essential part of any echo sounding system is the transducer. This is the physical device that converts an electrical signal in the hardware to an outgoing pressure wave in the sea, and an incoming pressure wave to an electrical signal. For ordinary applications in fisheries research, the transducer has directional patterns of transmission and reception: its angular distribution of radiation, or transmitted energy, is non-uniform, and its sensitivity to incoming radiation is similarly non-uniform.

These angular distributions are generally characterized by the so-called transmit and receive beam patterns. For computational convenience these quantities are generally computed as though the transducer operation were perfectly harmonic. This implies the fictions that the transmitted waveform is infinitely long and that the incoming waves have the same one-frequency character.

Assumption of single-frequency operation is undoubtedly excellent for the rather long pulse trains that are typically used in the surveying application (Johannesson and Mitson 1983, Mitson 1984). However, apropos of the universal tendencies in scientific instruments to greater precision and greater range of operation, it is worthwhile to examine critically the assumption.

It is the present ambition to develop an expression for an intrinsically wideband beam pattern, then make use of this in a theoretical investigation of the expected performance of several new transducers designed for the SIMRAD EK500 scientific echo sounder system (Bodholt et al. 1988, 1989). The computations are performed only for standard operating conditions, namely transducer resonance frequency of 38 kHz and nominal transmit pulse duration of 1 ms. Application of the definition of wideband beam pattern to other operating conditions or to other transducers is mentioned.

THEORY

The primary, and most elementary, question to be asked is what is meant by beam pattern. Conventionally this is described as the angular distribution of transmitted radiation in the farfield of the transducer or as the relative sensitivity of the transducer to the direction of incoming plane waves, all at the basic resonance frequency of the transducer.

For a planar transducer composed of n discrete elements, each with amplitude-weighting factor w_j, the two beam patterns are defined thus:

$$b(\theta,\phi) = \frac{\left| \sum_{j=1}^{n} w_j f_j(\theta,\phi) \right|^2}{\left| \sum_{j=1}^{n} w_j f_j(0,0) \right|^2} \quad , \qquad (1)$$

where f_j is the respective transmit or receive beam amplitude of the j-th element in the direction (θ,ϕ), relative to the axial direction $\theta=0$. If inter-element effects are neglected, then the transmit and receive beam patterns associated with a single element are the same, namely

$$b_j(\theta,\phi) = |f_j|^2 \quad , \qquad (2)$$

where

$$f_j(\theta,\phi) = A_j^{-1} \int_{A_j} e^{i\underline{k}\cdot\underline{r}} dA \quad , \qquad (3)$$

where A_j is the area of the j-th element, $\underline{k}$ is the wavevector, $\underline{k}=(2\pi/\lambda)\hat{k}$, λ is the acoustic wavelength, $\hat{k}$ is the unit vector in the direction (θ,ϕ), and $\underline{r}$ describes the position of the differential element of area dA on the transducer surface. If a Euclidean system is established with origin on the transducer and z-axis aligned with the acoustic axis or direction of maximum sensitivity, then $\hat{k}=(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ and $dA=dx\,dy$. Naturally, this implies that the transducer operation is of the single-frequency type.

The general inadequacy of this conventional formulation is evident from a very simple consideration. Since the transmit pulse is finite in duration, it has a spectrum or continuum of frequencies associated with it. The angular distribution of each of these may be described by equations (1) and (2), but these will be different since the acoustic wavelength λ, thence wavevector $\underline{k}$, varies with frequency.

Generalization of the single-frequency formulation follows that of the wideband backscattering cross section introduced in 1981 (Foote et al. 1981). A linear system model is appropriate (Foote 1982). Two cases are, however, distinguished.

Transmission What gets into the water depends both on the electrical signal that is sent to the transducer and also on the frequency response of the transmitter and coupled transducer, which may distort the intended transmit signal even before the frequency-selective effects suggested by equation (3) can operate. The intended transmit signal in the time domain, s(t), can also be described by its Fourier transform,

$$S(\omega) = \int_{-\infty}^{\infty} s(t)\, e^{-i\omega t}\, dt \quad , \tag{4}$$

where ω is the angular frequency, $\omega=ck$, and c is the medium speed of sound. The frequency response function of the transmitter and coupled transducer may be characterized by the impulse response function h(t) or Fourier transform $H(\omega)$. These two functions modulate the intrinsic frequency dependence of transducer response, as described by the single-frequency beam pattern in equations (2) and (3). If this is represented by the spectral amplitude $F(\omega)$, then the frequency response of the composite transmitting system is just the product $S(\omega)\,H(\omega)\,F(\omega)$. The angular distribution of energy in the transducer farfield is derived by analogy with the mentioned case of wideband backscattering cross section (Foote 1982), hence

$$b(\theta,\phi) = \frac{\int_0^{\infty} |S(\omega)\, H(\omega)\, F(\omega)|^2\, d\omega}{\int_0^{\infty} |S(\omega)\, H(\omega)|^2\, d\omega} \quad , \tag{5}$$

which makes use of the symmetry $X(\omega)=X(-\omega)$ in limiting integration to the domain $[0,\infty)$. The normalization assures the equality of wideband and single-frequency expressions in the limit of very long transmit pulse duration and negligible distortion of the transmit signal, for which $S(\omega)=\delta(\omega-\omega_o)$ and $h(t)=\delta(t)$, respectively.

Reception The receive beam pattern is likewise due to more than just the transducer acting alone as a vibrating geometric surface. It is similarly affected by the nature of the incoming signal, i.e., its spectral composition, and by the inevitable distortions, both wanted and unwanted, that are introduced by the electronics in the course of receiving the signal. If the spectrum of the incoming signal is represented by the function $S(\omega)$ and the frequency response function of the receiver by $H(\omega)$, which may be different from those defined for the transmission process, the receive beam pattern may be defined in exact analogy with equation (5).

COMPUTATIONAL MODEL

Performance measures

In addition to the transmit and receive beam patterns themselves, two useful measures of directivity are the receiving directivity index for isotropic noise and the directivity index for volume reverberation. The first depends on the integral $\int b(\theta,\phi)\, d\Omega$, where b pertains to reception. According to Urick (1983), the receiving directivity index for isotropic noise is

$$DI = 10 \log \frac{4\pi}{\int b(\theta,\phi)\, d\Omega} \quad . \tag{6}$$

The directivity index for volume reverberation depends on the integral $\int b^2(\theta,\phi)\, d\Omega$, where both transmitting and receiving functions are involved. By analogy with equation (6), the directivity index for volume reverberation is

$$J_v = 10 \log \frac{4\pi}{\int b^2(\theta,\phi)\, d\Omega} \quad . \tag{7}$$

This is commonly alternatively expressed through the equivalent ideal solid angle beamwidth, or equivalent beam angle,

$$\psi = \int b^2(\theta,\phi)\, d\Omega \quad . \tag{8}$$

These two quantities are now computed for several transducer configurations.

Transducers

Two transducers are considered. The first is the SIMRAD ES38B transducer, which is meant to be the standard 38-kHz, split-beam transducer for use with the EK500 scientific echo sounder. Its basic configuration is 10×10, with identical square elements of side length 30 mm and center-to-center distance along rows and columns of 32 mm. Weighting of the elements in the amplitude domain is described in Fig. 1.

0	0	70	100	100	100	100	70	0	0
0	70	70	100	100	100	100	70	70	0
70	70	100	100	100	100	100	100	70	70
100	100	100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100	100	100
70	70	100	100	100	100	100	100	70	70
0	70	70	100	100	100	100	70	70	0
0	0	70	100	100	100	100	70	0	0

Fig. 1. Relative amplitude weights of elements in the SIMRAD ES38B transducer.

The second considered transducer is the SIMRAD ES5 transducer. This 38-kHz split-beam device is based on a design specifying two separate beamwidths and corresponding operating modes (Foote 1988). Transducer elements are square, with the same side lengths and spacings as for the ES38B. The narrow-beam part of the transducer is based on the 14×14 pattern, with amplitude weightings as indicated in Fig. 2. The wide-beam part of the transducer is due to the central core of 92 elements, also shown in the same figure.

0	78	78	100	100	100	100	100	100	100	100	78	78	0
78	36	36	36	36	78	100	100	78	36	36	36	36	78
78	36	36	36	100	100	100	100	100	100	36	36	36	78
100	36	36	100	100	100	100	100	100	100	100	36	36	100
100	36	100	100	100	100	100	100	100	100	100	100	36	100
100	78	100	100	100	100	100	100	100	100	100	100	78	100
100	100	100	100	100	100	100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100	100	100	100	100	100	100
100	78	100	100	100	100	100	100	100	100	100	100	78	100
100	36	100	100	100	100	100	100	100	100	100	100	36	100
100	36	36	100	100	100	100	100	100	100	100	36	36	100
78	36	36	36	100	100	100	100	100	100	36	36	36	78
78	36	36	36	36	78	100	100	78	36	36	36	36	78
0	78	78	100	100	100	100	100	100	100	100	78	78	0

Fig. 2. Relative amplitude weights of elements in the two-mode SIMRAD ES5 transducer.

Transmit and receive signals

The transmit signal and receive signal, or signal in the medium before reception, are assumed to be the same square-wave-modulated sinusoid:

$$s(t) = \cos \omega_o t \ \mathrm{rect}\ \frac{t}{T} \quad , \qquad (9a)$$

where ω_o is the angular frequency, which is 2π times the ordinary frequency in Hertz, and T is the pulse duration. The signal spectrum is the Fourier transform of this, namely

$$S(\omega) = \frac{\exp[i(\omega-\omega_o)T/2]}{(\omega-\omega_o)T/2} \quad , \qquad (9b)$$

where only the positive part of the spectrum is shown, since $S(\omega)=S(-\omega)$.

Frequency response functions

Transmitter The combined transmitter-transducer system is represented by a pair of bandpass filters in series. These have the same characteristic response function,

$$H_j(\omega) = \frac{1}{1+iq_j\Delta} \quad , \qquad (10a)$$

where $i=\sqrt{-1}$, $\Delta=\omega/\omega_o-\omega_o/\omega$, but with $q_1=3$ for the transmitter and $q_2=8$ for the transducer.

Receiver The transducer-receiver system is represented by three bandpass filters in series, followed by a demodulator with two low-pass filters. The bandpass filters have the same characteristic function as in equation (10a), with $q_3=8$ for the transducer and $q_4=6.4$ for the first of the receiver bandpass filters. Either of two filters is used for the second receiver bandpass filter, with $q_5=6.4$ or 99 as the selected bandwidth is wide or narrow. The effect of demodulation with low-pass filtering is represented by shifting upwards, by ω_o, the characteristic low-pass filter functions,

$$H_j(\omega) = \frac{1}{1+i\omega R_j C_j} \quad , \qquad (10b)$$

where $i=\sqrt{-1}$, and R_j and C_j are the respective resistance and capacitance of RC-circuits equivalent to the low-pass filters. Shifting the function is accompanied by substituting the absolute value $|\omega-\omega_o|$ for ω in equation (10b). The values of R and C for the two low-pass filters, numbered '6' and '7', respectively, are the following: $R_6=5$ kΩ, $C_6=3.3$ nF, $R_7=43.2$ kΩ, and $C_7=680$ pF.

Combined frequency response function For purposes of computing the receiver directivity index, the filtering is represented by the product function

$$H_R(\omega) = \prod_{j=3}^{7} H_j(\omega) \quad . \tag{11}$$

Both the transmit and receive functions are involved in the equivalent beam angle, hence

$$H_{TR}(\omega) = \prod_{j=1}^{7} H_j(\omega) \quad . \tag{12}$$

Single-frequency beam pattern

This is computed for the several transducer configurations according to equations (1)-(3). The assumed medium sound speed is 1470 m/s.

RESULTS

Computed measures of directivity are presented in the Table. These are essentially indistinguishable for corresponding single-frequency and wideband computations, hence only one set of numbers is shown.

Table. Directivity measures for two transducers, as defined in equations (6)-(8), together with the logarithmic expression of the equivalent beam angle, denoted Ψ.

Transducer/mode	DI(dB)	J_v(dB)	ψ(sr)	Ψ(dB)
ES38B	27.6	30.9	0.0102	-19.9
ES5/Wide beam	27.6	31.1	0.0097	-20.1
ES5/Narrow beam	30.4	34.2	0.0048	-23.2

DISCUSSION

Beam patterns and other directivity measures have been computed for the several transducer arrays and transmitter and receiver filter functions. The tabulated results are indistinguishable for both the wideband method of computation and the ordinary single-frequency approximation. Differences first appear at the 0.01-dB level of precision.

This result might have been anticipated, for analogous computations in scattering have shown a similar agreement between corresponding results from the two methods. Averaged theoretical wideband target strengths of standard spheres, for example, are insensitive to pulse duration within rather large limits (Foote and MacLennan 1984). That is, changes in a prominent component of the beam pattern, the signal spectrum in equation (5), have little effect on the computed directivity measures. Averaged theoretical wideband backscattering cross sections of swimbladdered fish are scarcely distinguishable from their single-frequency counterparts (Foote 1985).

These findings of insensitivity in derived wideband measures of directivity to certain spectral functions are also supported by comparison of corresponding measures for both narrowband and wideband bandpass filters in the receiver, with respective Q-factors of 99 and 6.4. No difference is observed.

Basically, the filtering is strong in the present cases, and the pulse duration, 1 ms, is very long compared to a period of oscillation at the center frequency, namely 26 μs. Thus the transmission and reception processes are intrinsically narrowband. Differences can hardly be expected for such benign combinations of instrument parameters.

Differences are, however, apparent in the beam patterns. Changes in the method of computation, from single-frequency to wideband, or in the filtering, are evident in the computed beam patterns. The "null" regions or minima are especially sensitive, for while perfect cancellation may occur for a particular direction (θ,ϕ), the same balance is upset when the wideband nature of the ensonification and sensing is recognized. The position of nulls at different frequencies do not coincide, thus a condition of destructive interference does not obtain. This observation agrees with what is experienced in practice: measurements of beam patterns seldom display nulls as deep as those that are computed.

All in all, the particular findings show that the single-frequency approximation is entirely adequate for ordinary work. However, in very special cases, e.g., those involving very short or amplitude- or frequency-modulated signals, or with untuned instruments, the formal apparatus introduced here, including definition in equation (5), may be useful for predicting or explaining measures of directivity.

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