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ACOUSTIC SAMPLING VOLUME VERSUS EQUIVALENT BEAM ANGLE

by

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ABSTRACT

Improvements in target strength measurement have shown the equivalent beam angle (EBA) to be a worrisome source of error in echo integrator surveys. The reason is that the EBA is generally defined as a purely physical characteristic of the transducer. In fact, it depends on both the beam pattern and the detectability of fish, hence target strength, behaviour, and detection threshold. This is clearly seen through the concept of acoustic sampling volume, which is herein defined explicitly and evaluated numerically in several examples. In order to preserve the EBA in the echo integration equation, the EBA is generalized. Several methods for measuring this quantity *in situ* are described.

RESUME: VOLUME D'ECHANTILLONNAGE ACOUSTIQUE ET ANGLE EQUIVALENT DU FAISCEAU

Les améliorations des mesures d'index de réflexion ont montré que l'angle équivalent du faisceau (EBA) constitue une inquiétante source d'erreur dans les évaluations par écho-intégration. La raison en est que l'EBA est généralement défini comme une caractéristique purement physique du transducteur. En fait, il dépend à la fois de la fonction de directivité et des possibilités de détection du poisson, donc son index de réflexion, son comportement et le seuil de détection. Ceci est clairement mis en évidence par le concept de volume acoustique d'échantillonnage qui est ici défini en détail et évalué numériquement pour plusieurs exemples. Pour maintenir l'EBA dans l'équation de l'écho-intégration, on en donne une formulation simple. Plusieurs méthodes pour mesurer ce paramètre in situ sont décrites.

INTRODUCTION

The concept of acoustic sampling volume in fisheries acoustics is as old as the techniques of echo counting and integration. Whether or not the matter is addressed explicitly, it is always addressed, for fish density is measured by relating an observed quantity of fish to a volume, to wit, the sampling volume.

This volume is known to depend on the backscattering characteristics of observed fish as well as those of the observing transducer (Forbes and Nakken 1972, Yudanov and Kalikhman 1981, Kalikhman et al. 1981, Aglen 1983, Kalikhman and Tesler 1983, Ona 1987). It is also recognized to depend on the minimum detectable signal level or so-called detection threshold. The problem of determining the sampling volume is known alternatively as the threshold problem.

Ona (1987) has illustrated the very practical importance of the problem by the example of the target strength of cod at 38 kHz. The value, or target strength regression, applied in Norwegian echo integration surveys is significantly lower than that obtained from a wide variety of separate determinations, which are consistent among themselves. The cause is attributed to disregard of the threshold effect when the applied value was originally determined. Consideration of the sampling volume, through the equivalent beam angle, as depending solely on the transducer directivity resulted in an artificially low value of mean target strength.

Concern about the problem is not confined to this example. In fact, one of the recommendations of the Working Group on Fisheries Acoustic Science and Technology at its meeting in Ostende, 20-22 April 1988, is to meet in 1989 to consider, among other topics, the "effect of threshold on the conversion factor used in echo-integrator surveys", i.e., the problem of sampling volume.

Several different approaches to the problem are described in the literature. Computations are fewer, however. There is also a lack of succinct but comprehensive expressions for the sampling volume or threshold effect.

A notable, overtly statistical approach to a different but related problem is that by Weimer and Ehrenberg (1975). For a given threshold, the effect on a distribution of target strengths is described by an integral. This is evaluated numerically for a specific normal target strength distribution for each of several thresholds.

The present approach is physical. The sampling volume is described as earlier (Foote 1979) by a simple integral in which the dependences on fish position, backscattering cross section and orientation, or behaviour, in addition to those of transducer beam pattern and threshold, are shown explicitly. For applications to the echo counting and integration techniques of estimating fish density, an effective equivalent beam angle is defined. This is evaluated numerically for single point-scatterers, layers of point-scatterers, and fish, as represented by measured target strength functions of tilt angle at 38 kHz.

THEORY

A transducer is used both as transmitter and receiver. Its directional characteristics are contained in the one-way beam pattern b. A received echo is registered if its strength exceeds a minimum signal level or threshold t. The received echo strength is expressed as the product of a gain or geometric factor g, product of transmit and receive beam patterns b^2, and backscattering cross section σ.

For constant σ the sampling volume V_s is a fraction of the total available or accessible volume V_o:

$$V_s = \int_{V_o} H(gb^2\sigma - t)\, dV \quad . \tag{1}$$

The integrand H(x) is just a counting function, known formally as the Heaviside step function. This assumes the value 0, 1/2 or 1 as the argument x is less than, equal to or greater than 0. Thus for echo strengths $gb^2\sigma$ exceeding t, the contribution is registered.

Commercially important fish at ultrasonic frequencies are directional scatterers, and σ generally varies with orientation. To account for this in V_s, the integration in Equation (1) is also performed over the range of orientations determining the sampled values of σ according to the probability distribution function F. Thus

$$V_s = \iint H(gb^2\sigma - t)\, dF\, dV \quad . \tag{2}$$

This is tantamount to Equation (7) in Foote (1979), although with differences in nomenclature. For the case of constant σ, the integration over dF yields unity and Equation (1) results.

This expression for V_s is complete and unambiguous. However, its incorporation in echo counting and integration schemes requires adapting the equivalent beam angle ψ_o, which is defined entirely in terms of the transducer beam pattern (Simmonds 1984),

$$\psi_o = \int b^2\, d\Omega \quad . \tag{3}$$

Since this applies at a constant, farfield range, and $dV = r^2 dr\, d\Omega$, the solid-angle analogue to Equation (2) is

$$\iint H(gb^2\sigma - t)\, dF\, d\Omega \quad .$$

Comparing this with Equation (3), it is clear that the effective equivalent beam angle is

$$\psi = \iint b^2 H(gb^2\sigma - t)\, dF\, d\Omega \quad . \tag{4}$$

This quantity can, in one sense, be regarded as a generalization of the equivalent beam angle defined in Equation (3). However, its origin is in the concept of sampling volume, described in Equation (2). In fact, when

ψ is multiplied by $r^2\Delta r$, the product is equal to the sampling volume within a spherical shell of infinitesimal thickness Δr.

The gain factor g in the several equations is exemplified by two extreme, but not uncommon, situations of detection in the usual farfield of the transducer. (1) For a single scatterer, $g=10^{-\alpha r/5}r^{-4}$, where α is the coefficient of absorption given in decibels per meter, and r is the range in metres to the scatterer. (2) For a layer of identical scatterers, $g=10^{-\alpha r/5}r^{-2}$.

The detection threshold t has the same units as the product $gb^2\sigma$. At the very threshold, detection occurs essentially on the acoustic axis, where b=1. The scatterer, if directional, is in its most favorable aspect, where $\sigma=\sigma_{max}$. Here also the detection range is a maximum, and the gain factor g is a minimum. Thus $t=g_{min}\sigma_{max}$. In the limit that t vanishes, or the signal-to-noise ratio becomes very large, $V_s\to V_o$ and $\psi\to\psi_o$.

COMPUTATIONAL MODEL

In order to illustrate Equation (4), its various terms must be assigned numerical values. This is done through the following model.

Medium. This consists of sea water of salinity 35 ppt and temperature 5°C. The sound speed is thus 1470 m/s (Mackenzie 1981). At 38 kHz, therefore, the absorption coefficient α is 0.0106 dB/m (Francois and Garrison 1982).

Transducer. For convenience, the transducer is assumed to be circular, with half-beamwidth of 4 deg or full beamwidth between opposite 3-dB levels of 8 deg. The beam pattern thus depends only on the polar angle θ, and $b=[2J_1(ka \sin \theta)/(ka \sin \theta)]^2$, where $ka=1.61374/\sin(\pi/45)\doteq 23.1$. Performance of the integration in Equation (3) yields the nominal equivalent beam angle $\psi_o=0.0108$ sr or -19.66 dB.

Fish backscattering cross section. The source of data is the study by Nakken and Olsen (1977). Tabulation of their measurements of the tilt angle dependence of the dorsal aspect target strength function of cod (Gadus morhua) at 38 kHz (Foote and Nakken 1978) was drawn on in compiling a sample of eight functions. These were chosen for differences in specimen length by about 10 cm over the approximate range 10-90 cm. The backscattering cross section σ of tilt angle θ' is derived from the target strength value $TS(\theta')$ by the definition $TS=10 \log \sigma/4\pi$ (Urick 1975), but with use of SI units.

Fish behaviour. This is characterized in the usual way by a normal probability density function of tilt angle, $N(\overline{\theta'},s_{\theta'})$. Two sets of parameters are used: $(\overline{\theta'},s_{\theta'})=(0,5)$ and $(-4.4,16.2)$ deg. The empirical bases of the two sets are described in Foote and Ona (1987) and Olsen (1971), respectively. The tilt angle distribution is assumed to be truncated at two standard deviations from the mean. Thus the probability density function f in $dF=f\, d\theta'$ is $f\doteq 0.95^{-1}\exp[-(\theta'-\overline{\theta'})^2/2s_{\theta'}^2]$.

NUMERICAL METHOD

The integration in Equation (4) is effected in the following way. For the range r, less than the maximum detection range r_{max}, the equation $gb^2=t$ is solved for θ. Specifically, the equation

$$\frac{2J_1(ka \sin \theta)}{ka \sin \theta} = 10^{-\alpha(r_{max}-r)/20}(r/r_{max})^q$$

is solved numerically, where q=1 for a single point scatterer and q=1/2 for a scattering layer. The solution, denoted θ_r, is then used to limit the θ-integration in Equation (4), for the target at r cannot be detected anywhere outside the cone $\theta=\theta_r$.

Equation (4) is evaluated in the following discrete version:

$$\psi_r = 2\ \Delta\theta\ \Delta\phi \sum_{i=1}^{n_i} b^2(\theta_i) \sin \theta_i \sum_{j=1}^{n_j} \left\{ \frac{\sum_{k=1}^{n_k} H[b^2(\theta_i)\frac{\sigma(\chi_{ijk})}{\sigma_{max}} - \frac{g_{min}}{g_r}] f'(\theta_k')}{\sum_{k=1}^{n_k} f'(\theta_k')} \right\} , \quad (5)$$

where $\Delta\theta=\theta_r/n_i$,

$\theta_i=(i-1/2)\Delta\theta$,

$\Delta\phi= /n_j$,

$\phi_j=(j-1/2)\Delta\phi$,

$\Delta\theta'=4s_{\theta'}/n_k$,

$\theta_k'=\overline{\theta'}-2s_{\theta'}+(k-1/2)\Delta\theta'$,

$\chi_{ijk}=\pi/2 - \cos^{-1}(\sin \theta_i \cos \phi_j \cos \theta_k' - \cos \theta_i \sin \theta_k')$.

The subscript is attached to ψ to emphasize its applicability at range r. In the computations reported below, $n_i=20$, $n_j=6$, and $n_k=40$.

RESULTS

The effective equivalent beam angle ψ is examined first for a point scattererer and a layer of point scatterers. Equation (4), thence (5) too, is immediately simplified, for the scattering is independent of orientation, hence the integration over dF yields unity. Since b only depends on θ, integration over ϕ yields 2π. Given a maximum range of detectability, ψ is reduced to the following:

$$\psi_r = 2\pi \int b^2(\theta)\ H(b^2-g_{min}/g_r) \sin \theta\ d\theta \quad .$$

This, or rather its discrete version, analogous to Equation (5), is evaluated for r_{max}=400 m and the results presented in Table 1. Included are the maximum angle of detection θ_r and the logarithmic expression for ψ,

$$\Psi = 10 \log \psi \quad .$$

What is to be remarked on here, with force for the other computations too, is that an absolute comparison of the scattering strengths of the point scatterer and layer of point scatterers is not undertaken. Rather, each of two problems is examined, where each scatterer type has its detection threshold at 400 m. Under ordinary conditions, without this constraint, if the point scatterers in the layer were identical with the single point scatterer, the detection thresholds would of course be different.

The results for Ψ_r are compared with the respective results for the same scatterer at different maximum detection ranges in Table 2.

The effect of directionality in scattering by fish on Ψ_r is illustrated in Tables 3 and 4 for the single-scatterer case, hence with $g=10^{-\alpha r/5}r^{-4}$. The difference in the two tables is that of the assumed tilt angle distribution. The maximum detection range is assumed to be 400 m for each fish scatterer, independent of size and absolute target strengths.

DISCUSSION

Characteristics of ψ_r

A number of systematics expected from Equation (4) are confirmed by the computations. To elucidate these more strongly, the dependence on the backscattering cross section σ is essentially eliminated in the computations for Tables 1 and 2 by consideration of identical point scatterers. For these, the value of the product gb^2, when compared with the threshold value t, is decisive for determining whether an echo strength lies above or below t, hence is or is not detected. Since the so-called gain or geometric factor g decreases with increasing range, the maximum angle of detection, $\theta=\theta_r$ in the beam pattern b, also decreases with increasing r. This is evident in Table 1.

The numbers in Table 1, and those of the other tables too, show the expected monotonic decrease in θ_r with increasing r. In addition, ψ_r is seen to vanish at the maximum range r_{max} and to approach the nominal transducer value $\psi_o=0.0108$ sr, or -19.7 dB, asymptotically as r decreases.

Another systematic dependence seen in Tables 1 and 2 is the effect of scatterer type, single or layer, on ψ_r. The mechanism for this is the range dependence of g. For the same range $r>1$ m, g for the point scatterer is less than g for the layer of identical point scatterers, hence ψ_r for the point scatterer exceeds that for the layer. If this result seems contrary in the context of overall backscattering strength, it must be remembered that the maximum detection range is assumed to be the same for the two scatterer types. This assumption is artificial, for all things being equal, the layer of identical scatterers would be detected at a greater range than the single scatterer would be. However, it was not felt necessary to illustrate this fact here.

Comparison of the respective numbers in Table 2 also shows the effect of the maximum detection range r_{max} on ψ_r. To take a particular example, for a single point scatterer with r_{max}=500 m, ψ is within 1 dB of ψ_o for ranges less than 390 m, or 78% of r_{max}, while with r_{max}=200 m, ψ departs from ψ_o by 1 dB at 150 m, or 75% or r_{max}. The trend is very similar; the small difference is due to the absorption part of g, which does not scale with r in the same way as the spreading part does.

Having established and shown how the effective equivalent beam angle ψ_r varies for identical point scatterers, directional fish scatterers are now considered. Comparison of values for ψ_r in Table 3 with their single-scatterer counterparts in Table 1 shows that the effect of directionality in fish scattering is to decrease ψ_r below that of the point scatterer. In addition, the larger the scatterer and more directional the scattering pattern, the smaller ψ_r is for the same r, assuming identical values for r_{max}. This general trend is supported by internal comparisons of the numbers in each of Tables 3 and 4, although deviations are also to be observed. These reflect variations in scattering properties, especially with respect to scatterer orientation, that are intrinsic to the scattering process, but which are not so strong as to upset the described general trend.

The same considerations explain the differences between corresponding numbers in Tables 3 and 4. These are generally lower in Table 4, because the associated tilt angle distribution is so much broader than that assumed for Table 3 that the chance of sensing lower values of target strength is much greater for Table 4 than for Table 3. The only contrary example is that for the fish of 70 cm length, which illustrates both the variable nature of target strength and the importance of the mean angle too in determining the distribution of sensed target strength values. This is further illustrated by the value of ψ_r for the same fish for $r \geq 375$ m. Inspection of the source data in Foote and Nakken (1978) shows that the tilt angle corresponding to the maximum dorsal aspect target strength is roughly 16 deg. Both this and the main scattering lobe too lie outside the ±2 s.d. range of tilt angles for the distribution N(0,5) in Table 3, but are inside the range for the distribution N(-4.4,16.2) used in Table 4.

Future work, including *in situ* measurement

The mass of presented values for ψ_r in the tables must be reduced. At the same time, the data base for fish target strength can be extended. Allowances can also be made for differences in detectability due to size. These differences have been obscured, consciously, in Tables 3 and 4, as well as in Tables 1 and 2, by the convention of assuming constant r_{max}. Adjustment according to the form $t=g_{min}\sigma_{max}$ is straightforward.

Additional work to be undertaken includes these tasks or projects: (1) corroboration of the present results by *ex situ* or other controlled measurement, or perhaps use of the theory predictively to interpret *in situ* measurements of target strength - to test for consistency, and (2) incorporation of forthcoming summary results in the echo counting and integration techniques for estimating fish density.

This research plan and desired applications would be aided by *in situ* measurements. Two specific ways in which information can be gotten about the effective equivalent beam angle empirically are enumerated.

(1) By means of a dual-beam or split-beam echo-sounding system (Ehrenberg 1983), the angular positions of resolved single fish, as measured from the acoustic axis, can be observed. The change in distribution of detected angles with range - for an aggregation that is suitably dispersed in the water column - will indicate how ψ_r changes with r. Use of theory should permit quantification of this, hence derivation of the correct numbers to be used in the surveying application.

(2) Observation of target strength distributions may also accomplish in situ quantification of the effective equivalent beam angle. Examination of the change in form of target strength distribution with increasing depth - for the same suitably dispersed fish aggregation - may disclose the encroaching influence of the threshold at greater ranges. Use of theory, as by Weimer and Ehrenberg (1975) or in accordance with Equation (4), may achieve the desired quantification.

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Table 1. Effective equivalent beam angles, with maximum detection angle θ_r, for detection of a single point scatterer and a layer of point scatterers by a circular transducer with half-beamwidth 4 deg when the maximum detection range is 400 m.

	Point scatterer			Scattering layer		
r(m)	θ_r(deg)	ψ(sr)	Ψ(dB)	θ_r(deg)	ψ(sr)	Ψ(dB)
400	0	0	$-\infty$	0	0	$-\infty$
395	0.95	0.0008	-30.8	0.77	0.0006	-32.6
390	1.35	0.0016	-27.9	1.10	0.0011	-29.6
385	1.65	0.0023	-26.3	1.35	0.0016	-27.9
380	1.91	0.0030	-25.2	1.56	0.0021	-26.8
375	2.14	0.0037	-24.4	1.74	0.0026	-25.9
370	2.35	0.0042	-23.7	1.91	0.0030	-25.2
365	2.54	0.0048	-23.2	2.07	0.0035	-24.6
360	2.72	0.0053	-22.8	2.21	0.0038	-24.1
355	2.88	0.0057	-22.4	2.35	0.0042	-23.7
350	3.04	0.0062	-22.1	2.48	0.0046	-23.4
345	3.19	0.0066	-21.8	2.60	0.0050	-23.0
340	3.33	0.0069	-21.6	2.72	0.0053	-22.8
335	3.47	0.0073	-21.4	2.83	0.0056	-22.5
330	3.61	0.0076	-21.2	2.94	0.0059	-22.3
325	3.74	0.0079	-21.0	3.05	0.0062	-22.1
320	3.86	0.0081	-20.9	3.15	0.0065	-21.9
315	3.98	0.0084	-20.8	3.25	0.0067	-21.7
310	4.10	0.0086	-20.7	3.34	0.0070	-21.6
305	4.22	0.0088	-20.6	3.44	0.0072	-21.4
300	4.33	0.0090	-20.5	3.53	0.0074	-21.3
290	4.55	0.0093	-20.3	3.71	0.0078	-21.1
280	4.76	0.0096	-20.2	3.88	0.0082	-20.9
270	4.96	0.0098	-20.1	4.05	0.0085	-20.7
260	5.16	0.0100	-20.0	4.21	0.0088	-20.6
250	5.35	0.0102	-19.9	4.37	0.0091	-20.4
240	5.54	0.0103	-19.9	4.53	0.0093	-20.3
230	5.72	0.0104	-19.8	4.68	0.0095	-20.2
220	5.90	0.0105	-19.8	4.83	0.0097	-20.1
210	6.07	0.0106	-19.8	4.97	0.0098	-20.1
200	6.24	0.0106	-19.7	5.12	0.0100	-20.0
180	6.58	0.0107	-19.7	5.40	0.0102	-19.9
160	6.91	0.0107	-19.7	5.69	0.0104	-19.8
140	7.23	0.0108	-19.7	5.97	0.0105	-19.8
120	7.55	0.0108	-19.7	6.26	0.0106	-19.7
100	7.87	0.0108	-19.7	6.56	0.0107	-19.7
50	8.67	0.0108	-19.7	7.41	0.0108	-19.7

Table 2. Effective equivalent beam angles for detection of ideal scatterers by a circular transducer with 4-deg half-beamwidth for several maximum detection ranges r_{max}. The case r_{max}=400 m is repeated from Table 1.

$r_{max}-r$	r_{max} for single point scatterer				r_{max} for layer of point scatterers			
(m)	500	400	300	200	500	400	300	200
0	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
5	-31.2	-30.8	-30.0	-28.7	-33.1	-32.6	-31.9	-30.8
10	-28.5	-27.9	-27.1	-25.9	-30.0	-29.6	-28.9	-27.9
15	-26.9	-26.3	-25.5	-24.3	-28.4	-27.9	-27.3	-26.2
20	-25.7	-25.2	-24.4	-23.3	-27.2	-26.8	-26.1	-25.1
25	-24.9	-24.4	-23.6	-22.6	-26.3	-25.9	-25.2	-24.3
30	-24.2	-23.7	-23.0	-22.0	-25.6	-25.2	-24.6	-23.6
35	-23.7	-23.2	-22.5	-21.6	-25.1	-24.6	-24.0	-23.1
40	-23.3	-22.8	-22.1	-21.3	-24.5	-24.1	-23.6	-22.6
45	-22.9	-22.4	-21.8	-21.0	-24.1	-23.7	-23.2	-22.3
50	-22.5	-22.1	-21.5	-20.7	-23.8	-23.4	-22.8	-21.9
55	-22.3	-21.8	-21.3	-20.5	-23.4	-23.0	-22.5	-21.7
60	-22.0	-21.6	-21.1	-20.4	-23.2	-22.8	-22.2	-21.4
65	-21.8	-21.4	-20.9	-20.3	-22.9	-22.5	-22.0	-21.2
70	-21.6	-21.2	-20.7	-20.1	-22.6	-22.3	-21.8	-21.0
75	-21.4	-21.0	-20.6	-20.1	-22.4	-22.1	-21.6	-20.9
80	-21.3	-20.9	-20.5	-20.0	-22.2	-21.9	-21.4	-20.7
85	-21.1	-20.8	-20.4	-19.9	-22.1	-21.7	-21.3	-20.6
90	-21.0	-20.7	-20.3	-19.9	-21.9	-21.6	-21.1	-20.5
95	-20.9	-20.6	-20.2	-19.8	-21.8	-21.4	-21.0	-20.4
100	-20.8	-20.5	-20.1	-19.8	-21.6	-21.3	-20.9	-20.3
110	-20.6	-20.3	-20.0	-19.7	-21.4	-21.1	-20.7	-20.1
120	-20.4	-20.2	-19.9	-19.7	-21.2	-20.9	-20.5	-20.0
130	-20.3	-20.1	-19.9	-19.7	-21.0	-20.7	-20.3	-19.9
140	-20.2	-20.0	-19.8	-19.7	-20.8	-20.6	-20.2	-19.8
150	-20.1	-19.9	-19.8	-19.7	-20.7	-20.4	-20.1	-19.8
160	-20.0	-19.9	-19.7	-19.7	-20.6	-20.3	-20.0	-19.7
170	-20.0	-19.8	-19.7	-19.7	-20.4	-20.2	-20.0	-19.7
180	-19.9	-19.8	-19.7	-19.7	-20.3	-20.1	-19.9	-19.7
190	-19.9	-19.8	-19.7	-19.7	-20.3	-20.1	-19.8	-19.7
200	-19.8	-19.7	-19.7		-20.2	-20.0	-19.8	
250	-19.7	-19.7	-19.7		-19.9	-19.8	-19.7	
300	-19.7	-19.7			-19.8	-19.7		
350	-19.7	-19.7			-19.7	-19.7		
400	-19.7				-19.7			
450	-19.7				-19.7			

Table 3. Effective equivalent beam angles for simulated detection of resolved cod with tilt angle distribution N(0,5) deg by a circular transducer at 38 kHz with 4-deg half-beamwidth. r_{max}=400 m.

	Total fish length (cm)							
r(m)	8.0	20.5	30.0	40.0	50.0	61.0	70.0	89.0
400	-∞	-∞	-∞	-∞	-∞	-∞	-∞	-∞
395	-38.0	-41.8	-40.5	-45.0	-46.7	-44.4	-∞	-48.2
390	-32.9	-36.8	-35.8	-39.3	-41.2	-39.0	-∞	-42.1
385	-30.1	-33.7	-32.8	-36.1	-38.1	-35.8	-∞	-39.0
380	-28.1	-31.4	-30.7	-33.8	-35.4	-33.7	-∞	-37.0
375	-26.6	-29.8	-29.2	-32.1	-33.5	-32.2	-∞	-35.5
370	-25.6	-28.5	-28.2	-30.5	-32.2	-31.2	-57.7	-34.3
365	-24.8	-27.5	-27.4	-29.0	-30.6	-30.2	-45.0	-33.2
360	-24.1	-26.7	-26.7	-27.9	-29.6	-29.4	-40.8	-32.4
355	-23.6	-26.0	-26.1	-27.0	-28.8	-28.7	-37.8	-31.3
350	-23.1	-25.4	-25.6	-26.3	-27.8	-28.1	-35.2	-30.2
345	-22.7	-24.9	-25.2	-25.7	-27.2	-27.6	-33.1	-29.2
340	-22.4	-24.4	-24.9	-25.2	-26.6	-27.2	-30.9	-28.4
335	-22.1	-24.0	-24.6	-24.7	-26.1	-26.8	-29.5	-27.8
330	-21.8	-23.7	-24.3	-24.4	-25.8	-26.5	-28.3	-27.1
325	-21.6	-23.3	-24.0	-24.1	-25.4	-26.1	-27.5	-26.6
320	-21.4	-23.1	-23.8	-23.8	-25.0	-25.8	-26.8	-26.0
315	-21.2	-22.8	-23.5	-23.5	-24.8	-25.6	-26.3	-25.5
310	-21.0	-22.6	-23.4	-23.3	-24.5	-25.4	-25.8	-25.1
305	-20.9	-22.4	-23.2	-23.1	-24.2	-25.2	-25.4	-24.6
300	-20.8	-22.2	-23.0	-22.9	-24.0	-25.0	-25.0	-24.1
290	-20.5	-21.8	-22.7	-22.6	-23.7	-24.6	-24.3	-23.2
280	-20.4	-21.5	-22.4	-22.3	-23.3	-24.3	-23.7	-22.6
270	-20.2	-21.2	-22.2	-22.1	-23.0	-23.9	-23.0	-21.9
260	-20.1	-21.0	-22.0	-21.9	-22.8	-23.5	-22.3	-21.4
250	-20.0	-20.8	-21.9	-21.7	-22.6	-23.1	-21.8	-21.0
240	-20.0	-20.6	-21.7	-21.5	-22.4	-22.6	-21.4	-20.7
230	-19.9	-20.4	-21.6	-21.2	-22.2	-22.2	-21.0	-20.5
220	-19.8	-20.3	-21.5	-21.1	-22.1	-21.8	-20.7	-20.3
210	-19.8	-20.2	-21.4	-20.9	-21.9	-21.4	-20.4	-20.1
200	-19.8	-20.1	-21.3	-20.7	-21.8	-21.1	-20.2	-20.0
180	-19.7	-20.0	-21.1	-20.4	-21.5	-20.5	-20.0	-19.9
160	-19.7	-19.9	-20.9	-20.1	-21.1	-20.1	-19.8	-19.8
140	-19.7	-19.8	-20.7	-19.9	-20.7	-19.9	-19.7	-19.7
120	-19.7	-19.8	-20.6	-19.8	-20.3	-19.8	-19.7	-19.7
100	-19.7	-19.7	-20.4	-19.7	-20.1	-19.7	-19.7	-19.7
50	-19.7	-19.7	-19.7	-19.7	-19.7	-19.7	-19.7	-19.7

Table 4. Effective equivalent beam angles for simulated detection of resolved cod with tilt angle distribution N(-4.4,16.2) deg by a circular transducer at 38 kHz with 4-deg half-beamwidth. r_{max}=400 m.

	Total fish length (cm)							
r(m)	8.0	20.5	30.0	40.0	50.0	61.0	70.0	89.0
400	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
395	-43.2	-47.3	-44.6	-49.5	-53.7	-48.3	-47.0	-51.8
390	-38.0	-41.7	-39.8	-44.7	-47.1	-43.7	-41.3	-47.8
385	-34.9	-39.0	-36.5	-40.8	-42.8	-39.4	-38.8	-44.0
380	-32.8	-36.6	-34.0	-38.3	-40.6	-37.5	-36.9	-41.5
375	-31.1	-34.7	-32.4	-36.4	-38.6	-36.0	-35.6	-40.1
370	-29.8	-33.4	-31.2	-34.9	-37.3	-35.0	-34.7	-39.3
365	-28.8	-32.4	-30.4	-33.6	-36.1	-33.9	-33.9	-37.8
360	-27.9	-31.5	-29.8	-32.4	-34.7	-33.1	-33.1	-36.5
355	-27.1	-30.8	-29.2	-31.4	-33.5	-32.4	-32.5	-35.3
350	-26.4	-30.1	-28.7	-30.7	-32.8	-31.7	-31.8	-33.7
345	-25.8	-29.5	-28.3	-30.1	-32.1	-31.2	-31.1	-32.6
340	-25.3	-28.9	-27.9	-29.6	-31.7	-30.7	-30.1	-31.6
335	-24.9	-28.5	-27.5	-29.0	-31.3	-30.4	-29.3	-30.9
330	-24.5	-28.2	-27.2	-28.7	-30.8	-29.9	-28.4	-30.0
325	-24.1	-27.8	-26.9	-28.3	-30.3	-29.4	-27.8	-29.3
320	-23.8	-27.4	-26.7	-27.9	-30.0	-29.2	-27.2	-28.7
315	-23.5	-27.1	-26.5	-27.7	-29.6	-28.9	-26.7	-28.0
310	-23.2	-26.8	-26.3	-27.4	-29.3	-28.6	-26.3	-27.5
305	-23.0	-26.5	-26.1	-27.1	-29.0	-28.4	-25.9	-26.8
300	-22.8	-26.3	-25.9	-26.9	-28.8	-28.2	-25.5	-26.2
290	-22.4	-25.8	-25.4	-26.4	-28.4	-27.8	-24.9	-25.1
280	-22.1	-25.4	-24.9	-25.9	-28.0	-27.3	-24.2	-24.2
270	-21.8	-24.9	-24.5	-25.5	-27.6	-26.8	-23.6	-23.5
260	-21.6	-24.6	-24.0	-24.9	-27.4	-26.2	-23.0	-22.8
250	-21.4	-24.2	-23.6	-24.2	-27.2	-25.5	-22.5	-22.3
240	-21.2	-23.9	-23.3	-23.6	-27.0	-25.0	-22.1	-21.9
230	-21.1	-23.7	-23.1	-23.1	-26.8	-24.3	-21.8	-21.6
220	-20.9	-23.4	-22.8	-22.6	-26.6	-23.7	-21.5	-21.3
210	-20.8	-23.1	-22.5	-22.2	-26.3	-23.2	-21.2	-21.1
200	-20.6	-22.7	-22.1	-21.8	-25.9	-22.7	-21.0	-20.9
180	-20.4	-22.1	-21.4	-21.1	-24.6	-21.8	-20.7	-20.5
160	-20.2	-21.7	-20.9	-20.5	-23.0	-21.2	-20.4	-20.1
140	-20.1	-21.3	-20.6	-20.2	-21.8	-20.6	-20.1	-19.9
120	-20.0	-20.9	-20.3	-19.9	-20.9	-20.2	-19.9	-19.8
100	-19.9	-20.6	-20.1	-19.8	-20.3	-19.9	-19.8	-19.7
50	-19.7	-19.7	-19.7	-19.7	-19.7	-19.7	-19.7	-19.7