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CYLINDRICAL SONAR DESIGN FOR FISH ECHO SURVEYING

by

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ABSTRACT

Most applications of echo integration involve use of a fixed single-beam transducer that is rather narrow and downwards-oriented. It is often desired to sense fish in other directions, for example, to assess components of the fish stock which may be avoiding the surveying vessel. To assist in discussions of a general solution, a cylindrical sonar design is proposed. Such a sonar would be carried in a towed body, allowing detection and localization throughout the plane transverse to the vessel track. Beam patterns, equivalent beam angles and other measures of performance are computed for a range of frequencies and aperture sizes.

RESUME: ETUDE D'UN SONAR A TRANSDUCTEUR CYLINDRIQUE POUR LA PROSPECTION ACOUSTIQUE DES POISSONS

Dans le plupart des cas, l'écho-intégration est mise en oeuvre avec un transducteur fixe, à faisceau simple, plutôt étroit et orienté vers le bas. On veut parfois sonder dans d'autres directions, par exemple pour prendre en compte les composantes des stocks de poissons qui peuvent éviter le navire prospecteur. Pour alimenter une discussion sur une solution générale, l'étude d'un sonar à transducteur cylindrique est exposée dans cette communication. Un tel sonar serait mis en oeuvre dans un corps remorqué rendant possible la détection dans tout le plan perpendiculaire à la route du navire. Les fonctions de directivité et les angles équivalents du faisceau sont calculés pour toute une gamme de fréquences et d'ouvertures de faisceau.

INTRODUCTION

Towed bodies, also called towed vehicles, offer a superior platform for acoustic operation. Away from the towing vessel, the simple, hydrodynamically streamlined form may be very quiet indeed. However, to be useful, the body must be equipped, deployed, maneouvred, monitored as in tracking, retrieved, and serviced, and data must be collected, and stored or transferred to the towing vessel. Nonetheless, towed bodies are used in echo surveying of fish stocks.

Acoustically instrumented towed bodies often carry a single, stationary, downwards-oriented transducer. Sometimes this can be rotated, either in a separate on-board operation or during deployment. Sometimes two transducers are used, one oriented upwards and the other downwards, for example. Other degrees of acoustic instrumentation exist, but on the whole, these tend to be rather simple.

Given that towed bodies, for all their cumbersomeness, are used as acoustic platforms, it behooves the user to get as much performance out of them as possible. Of course, exploitation is only possible with respect to a potential. It is the purpose of this work to begin addressing the question of how much acoustic performance can be packed into a towed body.

The answer can only be partial. In fact, it is given with respect to a cylindrical sonar composed of staves aligned on a right circular cylinder and steered in the transverse plane. The potential increase in sampling volume is profound, which is the justification for the following.

DEFINITIONS

Problem

The present aim is to determine the effects of array size and operating frequency on overall performance for a range of sonar designs. This is done for cylinder diameters from 20 to 60 cm, with lengths from one to three times the corresponding cylinder diameter. The range of investigated frequencies is 20-120 kHz.

Transducer

In order to assess overall sonar performance, it is desirable to minimize extraneous influences, such as those due to the transducing potential of individual array elements. For this reason, a similar resonance condition is assumed at all frequencies. In particular, the Q-value, or ratio of the resonance frequency to the width of the spectrum (Clay and Medwin 1977), is assumed to be constant, independent of frequency.

Each transducer is assumed to be robust, or capable of being driven at the threshold of cavitation. This depends on the operating depth and on the exact nature of the medium, including state of dissolved gas and particulate matter. For definiteness, this has been crudely lumped into the following expression for the cavitation threshold pressure, in atmospheres:

$$p_{cav} = 0.0076\ \nu^{1.45} \quad , \qquad (1)$$

where ν is the frequency in kilohertz. This approximates the mean of

"Esche's limiting curves" over the range 20-200 kHz, as given in Clay and Medwin (1977).

For convenience, each transducer element is assumed to be square. The side length a is determined by two constraints. Firstly, the center-to-center distance of adjacent transducers along the cylinder circumference should not exceed 0.6λ, where λ is the acoustic wavelength. Secondly, adjacent transducers should be separated by a minimum gap distance, g_{min}. These constraints are due to H. Bodholt, SIMRAD. If the equality is assumed, and the several distances are measured conservatively along the circumference, then

$$a = 0.6\lambda - g_{min} \tag{2}$$

defines the element size. The minimum gap distance, g_{min}, is assumed to be a constant 2 mm throughout this work.

Sonar

The object sonar is defined as a set of N individual staves which are aligned and evenly spaced along the wall of a right circular cylinder of radius R. The number of staves is required to be a multiple of four:

$$N = 4\left[\frac{\pi R}{2(a + g_{min})}\right] \quad , \tag{3}$$

where the brackets indicate that the enclosed quantity is to be replaced by the largest integer less than or equal to it. Thus N is the largest integer exactly divisible by four for which the staves are separated at least by the gap g_{min}. The actual spacing or gap distance g_1 is

$$g_1 = \frac{2\pi R}{N} - a \quad . \tag{4}$$

Each stave consists of M elements which span the length ℓ,

$$M = \left[\frac{\ell + g_{min}}{a + g_{min}}\right] \quad . \tag{5}$$

The actual gap distance g_2 is

$$g_2 = \frac{\ell - Ma}{M - 1} \quad . \tag{6}$$

The length is two, four or six times the cylinder radius.

All elements of the same stave are equally phased and weighted. The stave thus acts as a single, if extended, element, with maximum sensitivity in the transverse plane of the cylinder or, in the language of array theory, in the broadside direction.

There are as many apertures as staves. Each aperture consists of N/4 staves. These are steered or phased so that a plane wave normally incident on the center of the aperture is received as though acting simultaneously on each of the staves, thus simulating a planar aperture. The staves are weighted equally in amplitude.

BEAM PATTERN

A basic characteristic of any acoustic system is its beam pattern. This is also an essential ingredient in a number of performance measures. The farfield beam pattern of the cylindrical sonar is therefore calculated.

A Cartesian coordinate system is established with x- and y-axes in the horizontal plane, y-axis coincident with the cylinder axis, and base of staves in the x-z plane, defined by y=0. The position of the base of the j-th stave is $\underline{r}_j=(x_j,0,z_j)=R(\cos\chi_j,0,\sin\chi_j)$, where χ_j is measured in the ccw direction about the y-axis from the x-axis. The aperture of interest is arbitrarily chosen as the uppermost N/4 staves, where the angular position of the j-th stave is $\chi_j=\chi_o+j\Delta$, $\chi_o=\pi/4-\Delta/2$, $\Delta=2\pi/N$. Steering in the normal direction, along the z-axis or $\hat{z}$, is accomplished by applying the phase $\gamma_j=-kz_j$, where $k=2\pi/\lambda=2\pi\nu/c$ is the wavenumber, λ the wavelength, ν the frequency, and c the medium speed of sound. The beam pattern b is computed in the direction (θ,ϕ), defined by the wavevector $\underline{k}=k(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta)$:

$$b(\theta,\phi) = \left| \frac{D_2\sin(M\eta)}{M\sin\eta}\,\frac{4}{N}\sum_{j=1}^{N/4} D_{1j}e^{i(\underline{k}\cdot\underline{r}_j+\gamma_j)}\right|^2 , \qquad (7)$$

where $D_{1j}=H(\underline{k}\cdot\hat{n}_j)(\sin\alpha)/\alpha$, $\alpha=ka(\sin\theta\cos\phi\sin\chi_j - \cos\theta\cos\chi_j)/2$, $D_2=(\sin\beta)/\beta$, $\beta=ka\sin\theta\sin\phi/2$, $\eta=k(a+g_2)\sin\theta\sin\phi/2$. The factor $H(\underline{k}\cdot\hat{n}_j)$ is the Heaviside step function: $H(x)=0$ for $x<0$, $\frac{1}{2}$ for $x=0$, and 1 for $x>0$, and $\hat{n}_j$ is the unit normal to the j-th stave. This factor acts as a counting function: it counts the contribution of the j-th stave to $b(\theta,\phi)$ only for the forward direction with respect to the same stave.

In deriving this expression the directivity of a single, square element has been factored into the parts D_1 and D_2. The first depends on the stave number, because the element normal is $\hat{n}_j=(\cos\chi_j,0,\sin\chi_j)$. The second part is independent of j, for $\hat{n}_j\cdot\hat{y}=0$. Another simplification has been effected because of the regular spacing of elements along each stave. This allows exact summation, with the result $\sin(M\eta)/(M\sin\eta)$ for the amplitude.

The basics of array theory are given in Urick (1975).

PERFORMANCE MEASURES

In order to compare different array designs, a number of performance measures are examined. These include the relative source level, relative signal-to-noise ratio in the receiver, directivity index, and equivalent beam angle.

The principal dependences of the source level SL are the total transmitted power P and the directivity index DI of the transmit array (Urick 1975):

$$\mathrm{SL} \propto 10 \log P + \mathrm{DI} \quad . \tag{8}$$

The total transmitted power is proportional to the aperture area S_{ap} and the operating power level, as determined by the cavitation threshold:

$$10 \log P \propto 10 \log S_{ap} + 20 \log p_{cav} \quad . \tag{9}$$

Here, and throughout this study, the pulse duration is assumed to be constant. The directivity index is defined in the accepted fashion:

$$\mathrm{DI} = 10 \log \frac{4\pi}{\int b \, d\Omega} \quad . \tag{10}$$

This is the same for both transmit and receive modes.

The signal-to-noise ratio SNR in the receiver depends on the directivity of the receiver, its bandwidth, and the detection threshold. The detection threshold depends on the noise level in the receiver. Over the frequency range of interest, 20-120 kHz, this is dominated first by wind noise and then by thermal noise in the medium. As described in Urick (1975), the spectrum of wind noise decreases at about 5-6 dB/octave, while thermal noise increases at 6 dB/octave. Wind noise dominates thermal noise up to about 40-200 kHz, depending on the wind speed. For present purposes, the frequency dependence of the noise level and the detection threshold are ignored.

By assumption of constancy in transducer resonance condition or Q-value over the frequency range, the receiver bandwidth is proportional to the center frequency ν. Summarizing,

$$\mathrm{SNR} \propto \mathrm{DI} - 10 \log \nu \quad . \tag{11}$$

The sign of the bandwidth term indicates its negative influence on the received signal.

The equivalent beam angle Ψ is defined as usual (Foote 1987):

$$\Psi = 10 \log \int b^2 d\Omega \quad . \tag{12}$$

COMPUTATIONS AND RESULTS

The several performance measures have been computed for each of five cylinder diameters: 20, 30, 40, 50, and 60 cm; for each of three array or stave lengths, corresponding to one, two or three diameters. The computations have been repeated at each of six frequencies: 20, 38, 50, 70, 100, and 120 kHz.

The integrations in Equations (10) and (12) have been performed numerically by means of the computer routine D01FCF (NAG Library 1984), as in Foote (1987). In keeping with the assumption of baffling of staves by the sonar housing, the effective limits of integration in θ are 0 and $3\pi/4$. By symmetry, the integration in ϕ is performed over the quadrant $[0,\pi/2]$, and the result is quadrupled. Other constraints on the integration are that the relative accuracy be better than 0.01, and that the beam pattern in Equation (7) be evaluated at at least 1000 points over the integration octant. Only in two cases is it necessary to increase the number of points.

For comparison purposes, the performance measures are related to the current SIMRAD split-beam transducer at 38 kHz. This consists of 68 circular elements of 35 mm diameter, with total active area 654.2 cm^2. At the sound speed assumed throughout this study, namely 1470 m/s, which applies to sea water of salinity 35 ppt and temperature 5°C (Mackenzie 1981), the directivity index is 27.79 dB and the equivalent beam angle is -20.24 dB (Foote 1988).

In order to compute the relative source level, the cavitation threshold pressure is related to that at 38 kHz through Equations (1) and (9). For frequency ν in kilohertz, the relative power level, ΔP_{cav}, is

$$20 \log \frac{p_{cav}(\nu)}{p_{cav}(38)} = 29 \log \frac{\nu}{38} \quad .$$

This is tabulated for the six frequencies.

ν(kHz)	20	38	50	70	100	120
p_{cav}(atm)	0.6	1.5	2.2	3.6	6.1	8.0
ΔP_{cav}(dB)	-8.1	0	3.5	7.7	12.2	14.5

The results are presented in Table 1. Both the directivity index DI and equivalent beam angle Ψ are first computed as absolute quantities. These are also related to the reference transducer, as are the source level SL and signal-to-noise ratio in the receiver, SNR.

DISCUSSION

Both the equations and the tabulated results support the following, expected systematics. At a fixed frequency, the larger aperture has the greater directivity index DI and smaller equivalent beam angle Ψ. For a fixed aperture, DI increases with frequency, while Ψ decreases. Approximate relations for the integrals in Equations (10) and (12) are

Table 1. Dimensions and performance measures of cylindrical sonars, assuming apertures composed of one quarter of N staves. Relative measures are expressed with respect to the current SIMRAD 38-kHz split-beam transducer. The medium speed of sound is assumed to be 1470 m/s.

2R (cm)	ℓ (cm)	ν (kHz)	a (cm)	g_1 (cm)	g_2 (cm)	N/4	M	NM	S_{ap} (cm^2)	DI (dB)	Ψ (dB)	ΔSL (dB)	ΔSNR (dB)	ΔDI (dB)	$\Delta\Psi$ (dB)
20	20	20	4.21	1.03	1.05	3	4	48	213	17.6	-12.0	-23.2	-7.4	-10.2	8.2
20	20	38	2.12	0.50	0.43	6	8	192	216	23.4	-17.8	-9.2	-4.4	-4.4	2.5
20	20	50	1.56	0.40	0.28	8	11	352	214	25.7	-20.1	-3.5	-3.3	-2.1	0.1
20	20	70	1.06	0.25	0.20	12	16	768	216	28.3	-23.0	3.4	-2.2	0.5	-2.7
20	20	100	0.68	0.24	0.24	17	22	1496	173	31.7	-25.9	10.3	-0.3	3.9	-5.7
20	20	120	0.54	0.21	0.21	21	27	2268	165	33.5	-27.4	14.2	0.7	5.7	-7.2
20	40	20	4.21	1.03	0.26	3	9	108	479	20.5	-14.9	-16.7	-4.5	-7.3	5.3
20	40	38	2.12	0.50	0.25	6	17	408	458	26.4	-20.8	-3.0	-1.4	-1.4	-0.5
20	40	50	1.56	0.40	0.27	8	22	704	428	28.7	-23.1	2.5	-0.3	0.9	-2.8
20	40	70	1.06	0.25	0.24	12	31	1488	418	31.9	-26.0	9.9	1.4	4.1	-5.8
20	40	100	0.68	0.24	0.21	17	45	3060	354	34.7	-28.9	16.4	2.7	6.9	-8.7
20	40	120	0.54	0.21	0.21	21	54	4536	331	36.5	-30.4	20.2	3.7	8.7	-10.2
20	60	20	4.21	1.03	0.44	3	13	156	691	22.3	-16.7	-13.4	-2.7	-5.5	3.6
20	60	38	2.12	0.50	0.29	6	25	600	674	28.1	-22.5	0.5	0.4	0.4	-2.2
20	60	50	1.56	0.40	0.21	8	34	1088	662	30.2	-24.9	5.9	1.2	2.4	-4.7
20	60	70	1.06	0.25	0.22	12	47	2256	634	33.3	-27.7	13.0	2.8	5.5	-7.5
20	60	100	0.68	0.24	0.20	17	68	4624	535	36.4	-30.6	19.9	4.4	8.6	-10.4
20	60	120	0.54	0.21	0.21	21	81	6804	496	38.2	-32.2	23.7	5.5	10.5	-12.0
30	30	20	4.21	0.50	0.95	5	6	120	532	21.8	-15.9	-15.0	-3.2	-6.0	4.3
30	30	38	2.12	0.24	0.20	10	13	520	584	27.4	-21.4	-0.9	-0.4	-0.4	-1.1
30	30	50	1.56	0.25	0.21	13	17	884	538	29.6	-23.7	4.4	0.6	1.8	-3.4
30	30	70	1.06	0.25	0.26	18	23	1656	465	32.5	-26.5	10.9	2.0	4.7	-6.3
30	30	100	0.68	0.22	0.21	26	34	3536	409	35.4	-29.4	17.8	3.4	7.6	-9.2
30	30	120	0.54	0.20	0.20	32	41	5248	383	37.8	-30.9	22.2	5.0	10.0	-10.7
30	60	20	4.21	0.50	0.44	5	13	260	1152	24.7	-18.8	-8.7	-0.3	-3.1	1.4
30	60	38	2.12	0.24	0.29	10	25	1000	1124	30.3	-24.4	4.9	2.5	2.5	-4.1
30	60	50	1.56	0.25	0.21	13	34	1768	1076	32.6	-26.7	10.4	3.6	4.8	-6.5
30	60	70	1.06	0.25	0.22	18	47	3384	951	35.4	-29.5	16.9	4.9	7.6	-9.3
30	60	100	0.68	0.22	0.20	26	68	7072	818	38.4	-32.4	23.7	6.4	10.6	-12.2
30	60	120	0.54	0.20	0.21	32	81	10368	756	40.2	-34.0	27.5	7.4	12.4	-13.7

Table 1. (cont.)

2R (cm)	ℓ (cm)	ν (kHz)	a (cm)	g_1 (cm)	g_2 (cm)	N/4	M	NM	S_{ap} (cm^2)	DI (dB)	Ψ (dB)	ΔSL (dB)	ΔSNR (dB)	ΔDI (dB)	$\Delta\Psi$ (dB)
30	90	20	4.21	0.50	0.31	5	20	400	1772	26.5	-20.6	-5.1	1.5	-1.3	-0.3
30	90	38	2.12	0.24	0.25	10	38	1520	1708	32.2	-26.1	8.6	4.4	4.4	-5.9
30	90	50	1.56	0.25	0.20	13	51	2652	1613	34.1	-28.4	13.6	5.1	6.3	-8.2
30	90	70	1.06	0.25	0.21	18	71	5112	1436	38.0	-31.3	21.3	7.5	10.2	-11.0
30	90	100	0.68	0.22	0.20	26	102	10608	1226	40.2	-34.2	27.3	8.2	12.4	-13.9
30	90	120	0.54	0.20	0.20	32	122	15616	1138	42.6	-35.7	31.7	9.8	14.8	-15.4
40	40	20	4.21	0.28	0.26	7	9	252	1117	24.5	-18.4	-9.1	-0.5	-3.3	1.8
40	40	38	2.12	0.30	0.25	13	17	884	993	29.8	-23.9	3.8	2.0	2.0	-3.7
40	40	50	1.56	0.28	0.27	17	22	1496	910	32.1	-26.2	9.2	3.1	4.3	-6.0
40	40	70	1.06	0.25	0.24	24	31	2976	836	34.9	-29.0	15.9	4.4	7.1	-8.8
40	40	100	0.68	0.22	0.21	35	45	6300	728	38.0	-31.9	22.8	6.0	10.2	-11.7
40	40	120	0.54	0.21	0.21	42	54	9072	661	39.5	-33.4	26.2	6.7	11.7	-13.2
40	80	20	4.21	0.28	0.25	7	18	504	2233	27.5	-21.4	-3.1	2.5	-0.3	-1.2
40	80	38	2.12	0.30	0.24	13	34	1768	1987	32.7	-26.9	9.8	4.9	4.9	-6.7
40	80	50	1.56	0.28	0.22	17	45	3060	1862	35.0	-29.2	15.2	6.0	7.2	-9.0
40	80	70	1.06	0.25	0.21	24	63	6048	1699	37.9	-32.0	21.9	7.4	10.1	-11.8
40	80	100	0.68	0.22	0.21	35	90	12600	1457	41.0	-34.9	28.8	9.0	13.2	-14.7
40	80	120	0.54	0.21	0.20	42	109	18312	1335	42.5	-37.5	32.3	9.7	14.7	-17.3
40	120	20	4.21	0.28	0.24	7	27	756	3350	29.2	-23.1	0.5	4.3	1.5	-2.9
40	120	38	2.12	0.30	0.24	13	51	2652	2980	34.6	-28.7	13.4	6.8	6.8	-8.4
40	120	50	1.56	0.28	0.20	17	68	4624	2813	36.8	-31.0	18.8	7.8	9.0	-10.7
40	120	70	1.06	0.25	0.21	24	95	9120	2562	39.7	-33.8	25.6	9.2	11.9	-13.6
40	120	100	0.68	0.22	0.20	35	136	19040	2201	42.7	-36.7	32.4	10.7	14.9	-16.4
40	120	120	0.54	0.21	0.20	42	163	27384	1996	44.2	-38.9	35.8	11.4	16.4	-18.6
50	50	20	4.21	0.70	0.37	8	11	352	1560	25.9	-20.3	-6.2	0.9	-1.9	-0.1
50	50	38	2.12	0.33	0.27	16	21	1344	1510	31.2	-25.8	7.1	3.5	3.5	-5.6
50	50	50	1.56	0.22	0.23	22	28	2464	1499	34.2	-28.1	13.5	5.2	6.4	-7.9
50	50	70	1.06	0.21	0.23	31	39	4836	1358	37.1	-31.0	20.2	6.6	9.3	-10.7
50	50	100	0.68	0.21	0.21	44	56	9856	1139	39.9	-33.8	26.7	7.9	12.1	-13.6
50	50	120	0.54	0.21	0.20	53	68	14416	1051	42.1	-35.4	30.9	9.4	14.4	-15.1

Table 1. (cont.)

2R (cm)	ℓ (cm)	ν (kHz)	a (cm)	g_1 (cm)	g_2 (cm)	N/4	M	NM	S_{ap} (cm^2)	DI (dB)	Ψ (dB)	ΔSL (dB)	ΔSNR (dB)	ΔDI (dB)	ΔΨ (dB)
50	100	20	4.21	0.70	0.35	8	22	704	3119	28.9	-23.3	-0.2	3.9	1.1	-3.1
50	100	38	2.12	0.33	0.21	16	43	2752	3092	34.9	-28.8	13.9	7.1	7.1	-8.6
50	100	50	1.56	0.22	0.23	22	56	4928	2998	37.2	-31.2	19.5	8.2	9.4	-10.9
50	100	70	1.06	0.21	0.21	31	79	9796	2752	41.5	-33.9	27.7	11.1	13.8	-13.7
50	100	100	0.68	0.21	0.20	44	113	19888	2299	42.9	-37.9	32.8	10.9	15.1	-17.7
50	100	120	0.54	0.21	0.20	53	136	28832	2102	44.5	-38.4	36.3	11.8	16.8	-18.1
50	150	20	4.21	0.70	0.21	8	34	1088	4821	30.7	-25.1	3.5	5.7	2.9	-4.9
50	150	38	2.12	0.33	0.23	16	64	4096	4602	36.4	-30.6	17.1	8.6	8.6	-10.4
50	150	50	1.56	0.22	0.20	22	85	7480	4551	39.6	-32.9	23.7	10.6	11.8	-12.6
50	150	70	1.06	0.21	0.20	31	119	14756	4145	41.9	-35.7	29.9	11.5	14.2	-15.5
50	150	100	0.68	0.21	0.20	44	170	29920	3459	44.8	-38.6	36.4	12.8	17.0	-18.4
50	150	120	0.54	0.21	0.20	53	204	43248	3153	46.3	-40.1	39.8	13.5	18.5	-19.9
60	60	20	4.21	0.50	0.44	10	13	520	2304	27.7	-21.9	-2.7	2.7	-0.1	-1.7
60	60	38	2.12	0.24	0.29	20	25	2000	2247	33.5	-27.4	11.0	5.7	5.7	-7.2
60	60	50	1.56	0.25	0.21	26	34	3536	2151	35.7	-29.7	16.5	6.7	7.9	-9.5
60	60	70	1.06	0.21	0.22	37	47	6956	1954	38.7	-32.5	23.3	8.2	10.9	-12.3
60	60	100	0.68	0.21	0.20	53	68	14416	1666	41.6	-35.4	30.0	9.6	13.8	-15.2
60	60	120	0.54	0.20	0.21	64	81	20736	1512	43.2	-37.0	33.6	10.4	15.4	-16.7
60	120	20	4.21	0.50	0.24	10	27	1080	4786	30.7	-24.9	3.4	5.7	2.9	-4.7
60	120	38	2.12	0.24	0.24	20	51	4080	4584	36.5	-30.4	17.2	8.7	8.7	-10.2
60	120	50	1.56	0.25	0.20	26	68	7072	4303	38.7	-32.7	22.5	9.7	10.9	-12.5
60	120	70	1.06	0.21	0.21	37	95	14060	3949	41.7	-35.6	29.4	11.2	13.9	-15.3
60	120	100	0.68	0.21	0.20	53	136	28832	3333	44.6	-38.4	36.0	12.6	16.8	-18.2
60	120	120	0.54	0.20	0.20	64	163	41728	3042	46.2	-40.0	39.6	13.4	18.4	-19.7
60	180	20	4.21	0.50	0.30	10	40	1600	7090	32.4	-26.7	6.9	7.4	4.6	-6.5
60	180	38	2.12	0.24	0.22	20	77	6160	6921	38.2	-32.2	20.7	10.4	10.4	-12.0
60	180	50	1.56	0.25	0.20	26	102	10608	6454	40.5	-34.5	26.1	11.5	12.7	-14.2
60	180	70	1.06	0.21	0.20	37	143	21164	5945	43.4	-37.3	32.9	12.9	15.6	-17.1
60	180	100	0.68	0.21	0.20	53	204	43248	4999	46.3	-40.2	39.6	14.3	18.5	-19.9
60	180	120	0.54	0.20	0.20	64	245	62720	4572	48.6	-41.7	43.8	15.8	20.8	-21.5

$$\left.\begin{array}{l}\int b\,d\Omega\\ \int b^2 d\Omega\end{array}\right\}\ \alpha\ \left\{\begin{array}{ll}(R\ell)^{-1} & \text{for constant frequency}\\ \nu^{-2} & \text{for constant size.}\end{array}\right.$$

For the sizes and frequencies considered here, the approximations are excellent. In fact, a small basis set of numbers, say those for $2R=20$ cm and $\ell=20$ cm, is sufficient to derive the values of the integrals, hence DI and Ψ, for the other values of R and ℓ. Deviations from the approximate relations, which are small, may be attributed to the effects of quantization. These are due to the constraints that N be exactly divisible by four and that the gap between adjacent transducer elements be at least 2 mm. Because of these conditions, the precise geometric configuration and computed properties do not scale exactly with the overall sonar dimensions.

The source level SL increases with both aperture size and frequency through the dependences given in Equations (8) and (9). The significant increase in cavitation threshold pressure with frequency, shown in Equation (1), is also represented in the tabulated numbers for ΔSL.

The signal-to-noise ratio, SNR, in the receiver also increases with aperture size and frequency because of its dependence on DI, in Equation (11). The increasing bandwidth of resonant transducer elements has a negative effect because of the admission of more noise into the receiver. This is why the increase in ΔSNR with increasing frequency is less rapid than that of DI.

Given the simple composition and interrelatedness of the present performance measures, Table 1 could have been abbreviated. However, in order to determine how much acoustic performance can be put into or gotten out of a towed body of fixed size, it was necessary to apply the design criteria consistently, hence respect the effects of quantization. Convenience was an additional consideration in preparing Table 1.

In a fuller analysis of performance, both the effect of medium absorption and detection threshold ought to be included. Such useful performance measures as the minimum detectable echo level and the figure of merit (Urick 1975) could then be computed.

Clearly, the rapid increase in absorption with increasing frequency will diminish the apparent advantages of operating at higher frequencies. For the design sound speed, 1470 m/s, the absorption coefficient α at the six frequencies is, according to Francois and Garrison (1982), the following:

ν(kHz)	20	38	50	70	100	120
α(dB/km)	3.8	10.6	15.2	21.5	28.5	32.2

Cost may also be a major factor in selecting a particular sonar design. That there is an optimization problem to be solved when performance and cost both have to be taken into account is recognized (Urick 1975, Hill 1986).

Two factors influencing this tradeoff in marine fisheries research are the expense of operating an ocean-going survey vessel and the risk of not observing fish at all for want of sufficient performance. This second condition has already motivated the proposal for an expanded split-beam transducer (Foote 1988). Such a transducer should aid both the echo-integration and target strength measurement of cod (*Gadus morhua*) and blue whiting (*Micromesistius poutassou*), not to mention redfish (*Sebastes marinus*).

As mentioned at the outset, this study only intends to begin addressing the problem of determining the acoustic potential of a towed body. To this end, the several performance measures have been referred to the current SIMRAD split-beam transducer at 38 kHz. What should not be forgotten in examining the relative performance measures is that the proposed cylindrical sonar can, by beam-forming, survey throughout the transverse plane. In the course of towing, therefore, the three-dimensional volume can be sampled to a much greater degree than can be done with a hull-mounted transducer, however sophisticated, i.e., powerful and sensitive.

Future work on designing a cylindrical sonar may involve a number of elements. Some of these are: (1) a fuller analysis of performance measures, including absorption and detection thresholds, (2) determination of how, hence where too, the signal processing ought to be performed, and (3) a cost analysis in which performance is optimized.

A fourth, and very interesting study, which is also related to the second, should determine strategies for sampling the volume. These might include automatic beam-steering according to sequences that are (i) operator-determined, (ii) random, as defined by some probability density function of angle in the transverse plane, (iii) systematic, as in sweeping continuously around the longitudinal axis, thus sampling the volume in a helical swath, and (iv) adaptive, in which a given beam-steering sequence may be interrupted and then determined by the detection of targets of sought characteristics, for a certain duration or until passed, when the original steering sequence may be resumed. The possibilities are only limited by imagination, for any system than can accomplish the processing will also be able to do the steering.

A fifth study might consider a general cylindrical array in which the beam can also be steered fore and aft. To do this, each individual transducer element would have to be independently controlled. Thus the use of staves would not be generally feasible, which would entail an order-of-magnitude increase in system complexity. The cost of this would have to be weighed against such benefits as being able to track individual targets or map the three-dimensional form of fish schools, e.g., herring (*Clupea harengus*).

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REFERENCES

Clay, C. S., and Medwin, H. 1977. Acoustical oceanography: principles and applications. Wiley, New York. 544 pp.

Foote, K. G. 1987. Dependence of equivalent beam angle on sound speed. ICES C.M./B:2, 6 pp. [mimeo]

Foote, K. G. 1988. Designing an improved transducer array geometry. ICES C.M./B:2, 8 pp. [mimeo]

Francois, R. E., and Garrison, G. R. 1982. Sound absorption based on ocean measurements. Part II: Boric acid contribution and equation for total absorption. J. acoust. Soc. Am., 72: 1879-1890.

Hill, J. 1986. The influence of operating frequency on transducer cost in sonar and echo sounder systems. Proc. 'First Australasian Port, Harbour & Offshore Engineering Conf.', Coll. Civ. Eng. Publ. by Institution of Engineers, Australia, ISBN 0-85825-311-9. Pp. 312-315.

Mackenzie, K. V. 1981. Nine-term equation for sound speed in the oceans. J. acoust. Soc. Am., 70: 807-812.

NAG Library. 1984. NAG FORTRAN Library Manual, Mark 11, Vol. 1, National Algorithms Group, Oxford.

Urick, R. J. 1975. Principles of underwater sound. Second edition, McGraw-Hill, New York. 384 pp.