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A METHOD FOR ESTIMATING THE EFFECTIVE MESH SIZES AND THE EFFECTS OF CHANGES IN GEAR PARAMETERS

BY
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page
3: Figure 1


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page 4: 1 19 3. top "and" instead of "of"
page 6: 1 21 top: Fig. }
page 7: 1 6 top: "too" instead of "to"
page 9: l 5 bottom: FL(e,L) = SL(e,L)RL(e,L)EF(e)
- - I 4 - the maximum fishing mortality
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page 11: 16 top: $\sum_{e=1}^{E} \operatorname{FLLAND}(e, L)$

- $\quad 119$ top: $\sum_{\mathrm{e}}$
- $\quad-110$ bottom: $\frac{\mathrm{dC}(\mathrm{T})}{\mathrm{dT}}=F(\mathrm{~T}) \mathrm{N}(\mathrm{T})$
page 12: 18 bottom: $C(e, T G(i+1))-C(e, T G(i))$
page 16: formula (6) $\sum_{e=1}^{E} \sum_{i}(\operatorname{CL}(e, i)-\operatorname{OBSCL}(e, i))^{2}$
    -         - $\quad 115$ top: $" F A C(e) L 50 \% "$ instead of $"=\mathrm{L} 50 \%=(\mathrm{FAC}-1) \mathrm{L} 50 \%$ "
-     - formula (6) $\sum_{e=1}^{E} \sum_{i}\left(\int_{T G(i)}^{T G(i+1)} \operatorname{DL}(e, L(t)) R L(e, L(t)) E F(e)\right.$
$\left.\underline{\exp \frac{[(L(t)-\operatorname{MESH}(e) \cdot \operatorname{SEL}(e)) \log (3))]}{(\operatorname{FAC}(e)-1) \operatorname{MESH}(e) \operatorname{SEL}(e)}} N(t) d t-\operatorname{OBSCL}(e, i)\right)\left.\right|^{2}$
$1+\exp \left[\frac{L(t)-\operatorname{MESH}(e) \operatorname{SEL}(e) \log (3)]}{(\operatorname{FAC}(e)-1) \operatorname{MESH}(e) \operatorname{SEL}(e)}\right.$
page 17: line 1 top: $\frac{\operatorname{dLAND}(2, t)}{\operatorname{dt}}=$
$\frac{\operatorname{dLAND}(E, t)}{d t}=$
        -             - line 3 top: "initial" instead of "a initial"
        -             - 8 bottom: $\frac{\operatorname{OBSCL}(e, i)}{\sum \operatorname{OBSCL}(e, i)}$
        -             - 6 bottom: $\frac{i}{\sum_{i}^{C L}(e, i)}$
_ - - 2 bottom: "two" instead of "to"
page 18: line 4 top: $(1 / \Sigma \operatorname{CL}(e, i))^{2}$



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SUMMARY:
This paper is a description of a mesh assessment method, which has been used by some ICES Working Groups in recent years. The method gives a way to estimate the effective mesh sizes in effect for different fleet units. Further the model used makes assessment possible on fleet level. Once the data base is established, simulations can be done to illustrate effects of excluding one fleet, changing the gear chararacteristics of one or more fleets etc.
The calculatory work has been reduced very much be implementation of the model on a Nord loo computer in Bergen and on an IBM- computer at NEUCC, Copenhagen. One version will be available on the Nord-loo computer at ICES Head quarters in early 1981.
The computer versions are available on request from C.J.Rörvik, Bergen and Per Sparre, Copenhagen.
Sections 1 to 3 deal with different aspects of the model, section 4
gives a formal description of step 1 , where the effective mesh size is estimated, and section 5 a formal description of step 2, where the effect of changes in gear or other parameters are assessed.
In Appendix A the robustness of the model is tested, and in Appendix B the methematical derivation of the selection curve used in the model is given.

## 1. Introduction

An important part of the work of the fisheries research has been to estimate the short-term and long-term effects on catches and stock caused by changes of the selectivity of the gear being used, in particular of changes in the mesh size of trawls. Models for such assessments has been studied be several authors (Gulland 1961, 1964; Jones 1961, 1974).

Besides assumptions inherent in the models these methods assume that the present mesh size is known. However, as Gulland (1961) points out it is often a practical problem to determine precisely the average effective mesh size in use. He mentions two factors that may contribute to make the effective mesh size of a trawl different from the minimum legal size:

1) Shrinkage of new nets and subsequent stretching.
2) Chafing gear.

We will add some other factors to the list:
3) Lining inside the codend by small mesh netting.
4) Clogging of the net by fishes, especially when large catches are taken. This might be a significant factor even for small (by-)catches of fishes like redfish or flatfish.
5) The selective properties might change as the towing speed is changed. Higher speed may make the meshes more elongated and cause a lower selection factor.
6) The (direct measureable) mesh size in the codend may be different from the minimum legal one.

To elaborate the second factor somewhat. A covernet if allowed might be of too small meshsize. For example, a covernet of the same mesh size as used in the codend (a double codend) is found to reduce the effective mesh size by about $20-30 \%$ (Sætersdal 1960). Tight ropes around the codend might reduce the effective mesh size. (Beltestad 1977).

To make the sixth factor more clear; the codend may have been made of a slightly larger meshsize than the minimum legal size in order to make sure to be on the "legal" side. Alternatively a codend with too small mesh sizes may be used illegally.

The present paper describes firstly a method that on the basis of the lengthcomposition, or the age-composition of the catches from fleets that exploit the same stock, gives an estimate of the effective mesh sizes in use by the fleets.

The basic idea of the model is to estimate for a given set of mesh sizes of the gears, the expected length- (or age-) distributions, and then to compare the estimated with the respective observed distributions. An optimization routine changes the mesh sizes in order to minimize the sum of the squared distances between the estimated and the observed relative length- (or age-) distributions.

When this sum of squares is at some defined minimum the optimization is terminated. Within the realism of the model and the quality of the fixed input parameters, we then have estimates of the effective mesh sizes used for the period from which the observed length- (or age--) distributions are taken.

A flow chart for these main ideas is given in Fig. 1.
In addition to the catch-composition data, information about how the availability of the fish to the different fleets changes as the fish grow is needed. Information on discard practise of small fishes is essential, as this practice,


Fig. 1. Outline
if used, makes the composition of the landed fish different from the actual catch composition. The method further requires information about the total fishing mortality and how it is distributed on the different fleets. The model uses the von Bertalanfy growth equation, to correct the length with age, and its parameters are needed in the input data.

For fleets that do not use trawl the gears are regarded as if they were trawls. The estimated effective "mesh sizes" for these "non-trawl" fisheries are regarded as comparable to those for trawl fisheries that generate the same length(or age-) distributions.

The results from this model may be of interest itself, not only because it gives estimates of the effective mesh sizes, but also because one might to some extent test the consistency of the basic input data versus independent information.

This model for mesh assessment is called STEP 1 in the present paper. The results from STEP 1 may be used in another model, STEP 2, which gives assessments of the effects on catches after a change in mesh sizes (or a change of any other parameter used in STEP 1). These effects are estimated on yearly basis until the new stability is achieved. (Within small modifications of the computer programs other time intervals might easily be used).

STEP 2 requires that the results are given in terms of catch of discards per age-group rather than per length-group. Assessments done on the basis of lengthdistributions may easily be recalculated in terms of age-groups for the same estimated effective mesh sizes.

The basic ideas behind these models (STEP 1 and STEP 2) were originally used by Mr. K. P. Andersen at the Danish Institute of Fisheries on data from the Faroe Working Group (Anon 1974). However, Andersen has not published the method. Hoydal (1977) described briefly part of STEP 1 and applied preliminary versions of the two methods to catch data from a stock of cod and a stock of haddock off the Faroe Islands.

In the first calculations, based on this method, the comparison between calculated and observed length distributions was done by eye. This was a very timeconsuming business, and left the decision to the subjective judgment of the person applying the method.

The method has later been generalized to some extent, and optimization routines have been applied by Rörvik in Bergen and by Sparre at the Danish Institute of Fisheries in Charlottenlund. During this work somewhat different computer programs have been developed in parallel at the two institutes. However, the programs have been tested against each other and have given consistent results within the accuracy of the different numerical approximation methods applied. The computer programs written by Rörvik were utilized by the Arctic Fishries Working Group (C.M. 1979/G:20), where also a brief description of the methods are given. The methods were also utilized by the Working Group on Redfish and Greenland Halibut in Region I (C.M. 1980/G:4).

These optimization routines have brought the time scale for application of the method down to the ones used by most Assessment Working Groups, and has further given a well defined and objective basis for judging, if the correspondence between observed and calculated distributions is satisfying.

The respective computer programs are available on request from Sparre and Rörvik. The methods described below apply to both sets of computer programs.

## 2. Chosing the input parameters.

## Observed length- or age-distributions

These observed values should be averaged for several years, so that the assumption of constant recruitment applies reasonable well. The number of years should be at least the same as is the number of fully exploited agegroups.

## The von Bertalanffy parameters

These include the assymptotic length as the age increases (L8), the growth rate (K) and the age at zero length (TO).

Keeping all the other parameters constant, the higher the $K$ (or L8) the lower becomes the effective mesh size when estimated from the length distributions. This is intuitively reasonable. The shorter time a yearclass spends in a length group, in particular the smallest length groups, the lower must the effective mesh size be, in order to "explain" the observed frequency of the smaller fishes in the catch. The effective mesh sizes are independent of the value of TO when using the length distributions.

Simulations on the basis of the age distributions show rather contrary effects. The estimates of the effective mesh size increase as L8 or $K$ increase. This is reasonable as the average length of any agegroup then increases. Thus higher effective mesh sizes are required, otherwise the youngest agegroups will be over-represented in the simulated age distributions. If TO increases the average length of any age group decreases, and by similar reasoning- a lower effective mesh size is required when using the observed age distributions.

A problem exists in that the model requires one set of growth parameters that should be representative for the whole stock. However, different fleets may fish on different components of the stock that may have different growth rates etc.

## Selection factor

This factor can be determined in mesh selection experiments. It depends on the fish species and its condition. The more elongated fishes have higher selection factors. It also depends on the matherial of the trawl. In the trawl, used in the selection experiments, is close to the one used in the actual fishery simulated, the factors 2,4 and 5 (page 1) are taken into account to some extent. This improves the so called "effective mesh size" as a measure of the real mesh size in use.

The models estimates the $50 \%$ selection point for each fleet ( $550 \%$ ). The corresponding mesh size is then readily calculated by dividing this figureby a selection factor determined as mentioned above.

Even if no estimates of selection factors are at hand, the relative size of the meshes in the different fleets is still valid, although it is not possible to relate them to the exact size of mesh.

## $75 \%$ 드웅

This ratio defines the steepness of the left part of the selection curve, as it gives the ratio, between the $75 \%$ and the $50 \%$ selection length of the gear. The $L 75 \% / L 50 \%$ - ratio is assumed to be constant and independent of the effective mesh size. As for the selection factor this ratio can be determined from mesh selection experiments:

## Recruitment function

The recruitment (availability) of the fishes to a particular fleet, as a function of length, has an ascending (recruiting) and descending (de-recruitment) part, of which one or both may be outside the length- (age-) range simulated.

A fleet might not cover the whole area of distribution for any size of the fish from the stock concerned. Therefore a proper interpretation of the recruitment function that varies between 0 and 1.0 is not the proportion of all the fishes of length $L$ that are available to the fleet concerned, but rather the availability at length $L$ compared with the maximum availability to the fleet of fishes of any length.

The parameters needed in the recruitment function are not as easily determined from experiments as the selection factor and the $L 75 \% / L 50 \%$ ratio. However, general knowledge about the distribution of the stock and its migration may help. We know for example from surveys and sampling of commercial catch that in most yearclasses of North-East Arctic cod, the young fish concentrates in the eastern part of the Barents Sea, and as it gets older it tends to concentrate more westerly and approach the spawning grounds. The general fit between the observed and the estimated length- (or age-) distributions may also give clues to whether the input parameter values are reasonable.

The recruitment function used affects the estimation of the effective mesh sizes in the following way; the more the ascending part is shifted to the right (Fig. ), that is higher values of RL50 \% and RL75 \%, the lower becomes the estimated effective mesh size. The less the smaller fish are assumed to be available, the higher selectivity of the gear for smaller fish is needed in order to explain the observed occurrence of these sizes in the catch. The position of the descending (de-recruitment) is not so critical for the estimation of the effective mesh sizes, although the de-recruitment parameters may be manipulated somewhat in order to get a better fit between the observed and estimated distributions.

For example, if the estimated frequencies drops much faster than the observed frequencies as the length (or age-) increases above that value giving the peak of the frequencies, it may mean that the de-recruitment function applied is too steep, or biased towards the lower length groups. However, the discrepancies may also be caused by a too high total mortality assumed, which leaves fewer fish to survive to the higher length- (or age-) groups.

## Discard parameters

Discarding of small fishes at sea cuts off the lower tail of the length- (and age-) distributions, and it is essential to have reliable observations about this practice. In the model only discarding of small fish is assumed. If however, discarding of large fishes occurs in a particular case, the model and the computer programs may easily be changed to take this into account.

The discard parameters affect the outcome of the simulation in the same general way as the recruitment parameters. The larger the length of discarded fish, the lower become the estimates of the effective mesh sizes.

## Fishing_Mortalities

The first problem is to choose the fishing mortality coefficient on the agegroups subject to maximum exploitation to put in to the simulations.

This corresponds to the summed fishing mortality by all fleetson the same agegroups.
One reasonable way to do this is to choose the corresponding values from VPA, averaged for the same years which the basic data in the mesh assessment cover.

Assuming a constant natural mortality coefficient, increases in the fishing mortalities, assumed in the simulations, will give higher estimates of effective mesh size, or put another way, to get correspondence between observed and simulated distributions, there has to be assumed a higher effective mesh size if the fishing mortality is increased. If this is not done the simulated distributions will have to large numbers of small fishes.

The second problem is to split the fishing mortality between fleets. If there are age- or length groups, where all fleets exploit the stock at maximum, the splitting is done according to the proportion of catches in numbers by each fleet in these groups.

There is another iterative way to estimate the split on fleets by "educated guesses" by the user until the estimate of the total catch, by number or weight, is distributed between the different fisheries in the same proportions as the observed catches. This alternative requires, however, several optimizations, especially if other input parameters, which affect the estimated catch distribution, are changed concurrently.

A third method is to change the estimated proportions $F$ until the estimated fishing mortality on agegroups subject to maximum exploitation becomes equal to the observed fishing mortality in each fleet. The observed fishing mortalities generated by each fleet might be calculated from VPA. However, this requires that the basic age compositions of the catch for each fleet are available for the years concerned.

## Natural mortality M

In the programs written the natural mortality coefficent, $M$, is assumed to be constant for all length- (and age-) groups. This has been done since it has been the usual practice in stock assessments. However, a lenght- (age-) dependent natural mortality curve could easily be included. If $M$ is changed, it should be recognized that the input fishing mortality on maximum exploited agegroups probably also should be changed, since it is often derived from methods (VPA or catch curves) where the calculated fishing mortalities depend on the assumed natural mortality.

## An example_of sensitivity

The sensitivity of the results to parameters used is rather variable. For example, the estimated effective mesh sizes are unsensitive to changes ir the discard curve on the recruitment curve when the ascending part of these curves are well to the left of the left part of the selection curve (as defined by the effective mesh size).

We have made a simple sensitivity analysis of the estimates of the effective mesh sizes for North-East-Arctic cod 1967-1977 (C.M. 1979/G:20). The sensitivity analysis is performed in the way that one or a set of input parameters in terms of cm rather than years (C.M. $1979 / \mathrm{G}: 20$, Table 26 ) are increased by $10 \%$, the others being held constant. The response is measured as the percentage change of the unweighted average of the new estimated effective mesh sizes compared with the unweighted average of the effective mesh sizes given by the Arctic Fisheries Working Group (C.M. 1979/G:20, Table 27). The results are given in Table 1 and they illustrate the general description of the interdependence given in the previous section. (Table 1 on page 25)

There are obviously possibilities to manipulate the input parameters in order to get low estimates of effective mesh, especially by shifting the parameters for the ascending recruitment curves or the discard curves towards higher length values.

However, accepting the observed length- (or age-) distributions there are upper limits to what the effective mesh sizes could be. Estimates of these maximum possible effective mesh sizes are achieved by using the most "conservative"
input parameters. This is examplified in the Arctic Fisheries Working Group where the maximum effective mesh sizes for North-East Arctic cod and haddock (C.M. $1979 / \mathrm{G}: 20$, Table 27) are given. In the report from the Working Group on Redfish and Greenland Halibut in Region I (C.M. 1980/G:4) the maximum mesh size of redfish on subareas is given. In these calculations the von Bertalanffy parameters and $M$ are accepted and left unchanged. But by using full recruitment from length (age) zero, no derecruitment and no discards a maximum effective mesh size is estimated.

## 3. USE OF THE METHOD

The use of these methods is not restricted to estimating effective mesh sizes and the yearly effects of a change in gear parameters or fleet parameters, although the present description of the methods are focused on these possibilities. Effects of a change in the discard practice, or a shift of a fishery to an area where the recruitment is different etc., may beestimated. The effects on the other fisheries of reducing or excluding one fishery may be estimated by reducing or set $F$ at zero for this particular fishery. Thus prognosis on the seperate fleets as well as for the total fisheries may be given. This has be done by Sparre (1980), and Hoydal (1977, 1980).

In the model the catch is calculated for a particular fleet, by using the product between the discard curve, the selection curve, and the recruitment curve. Except for the discard curve, the curves consist of an ascending and a descending part (Fig. 2 ). As described above all parts of the curves are fixed by the input parameters, except for the ascending part of the selection curve which depends on the effective mesh size which is to be determined.

However, any other parts of curves may be estimated. If, for example, the effective mesh size is determined by an independent method, the recruitment to the fisheries may be determined by an appropriate rearrangement of the input parameters. Or for example, the derecruitment could be estimated by fixing the selection curve and let the guessed length at $50 \%$ derecruitment replace the initial guess of the downward slope of the effective mesh size. If several of these different curves are simulated, one should be aware of the possibility of circular arguments.

Questions may be raised about the validity of the exact estimates of the effective mesh size. However, if one wants to investigate the effects of raising the legal mesh size by let say 35 mm , and one assumes this correspond to an increase of the effective mesh size by the same amount ( 35 mm ), then it has been our experience that these effects ( in \% change) are not very sensitive to the initial mesh sizes choosen, that is if one choose the old legal mesh size or the estimated effective mesh size. This means that STEP 2 used in the connection eith STEP 1 may be useful even when the results from STEP 1 are questionable, the relative effects will be less questionable.

## 4. FORMAL DESCRIPTION OF STEP 1

### 4.1 Fishing mortality at length and age.

In the present context the population to be considered is a yearclass during its lifespan.

Let
$N(T I)=$ the initial number of fish of the youngest age (TI years) considered.
$N(T)=$ the number of survivors at age $T$.
As Beverton and Holt (1957) we shall assume a constant recruitment, and constant mortality on each age- or length group, from year to year.

A corollary of this assumption is that the characteristics of a yearclass during its lifespan equal the characteristics of all agegroups of one particular year.

Consider one stock exploited by E fishing fleets (or fisheries). Within each fishing fleet all vessels are assumed to use the same type of gear.

A fishery is characterized by:

1. Selection curve:
SL (e, L)
2. Recruitment curve:
RL (e, L)
3. Discard curve:
DL (e, L)

Let $E$ be the number of fishing fleets.
SL (e, L) is the fraction retained of the fish of length $L$ entering the gear of fishing fleet no e. e = 1, 2, ...., E.
SL is composed of anascending and a descending part. For example the larger fish might not so easily get entangled in a gill net as the medium sized ones.
All fish that enter the gear, without being retained, are assumed to survive.
RL (e, L) is the fraction of the total number of fish of lenith $L$ available to the fishery of fleet $c$.
The recruitment is composed of an ascending and $e$ descending part (derecruitment) describing the migration of the fish into and out of the area exploited by fleet e.
DL (e, L) the fraction of the number caught of length $L$ which is not discarded. No discarded fish are assumed to survive.

Fishing mortality exerted by fishing fleet $e$ on the fish of length $L$, $F L(e, L)$ is the sum of the mortalities:

$$
\operatorname{FL}(e, L)=\operatorname{FLLAND}(e, L)+\operatorname{FLDISC}(e, L)
$$

Where FLDISC is the mortality caused by discarding and FLLAND is the remaining fishing mortality, the socalled "landing mortality".

Fishing mortality is assumed to be the product of three factors:

$$
F L(e, L)=S L(e, L) E F(e)
$$

Where $E F(e)$ is the fishing mortality on age or length groups exerted by fleet e. Discard and landing mortalities are:

$$
\begin{aligned}
& \operatorname{FLLAND}(e, L)=\operatorname{DL}(e, L) \quad F L(e, L) \\
& \operatorname{FLDISC}(e, L)=(1-\operatorname{DL}(e, L)) \operatorname{FL}(e, L)
\end{aligned}
$$




Fig. 2.. .The model of fishing mortality, as a function of length.

Figure 2: illustrates the various factors used in the expression of fishing mortality.

Total fishing mortality is the sum of the mortalities caused by the individual fleets.

$$
\begin{aligned}
\operatorname{FL}(L) & =\sum_{e=1}^{E} \operatorname{FL}(e, L) \\
\operatorname{FLLAND}(L) & =\sum_{e=1}^{E} \operatorname{FLLAND}(e, L) \\
\operatorname{FLDISC}(L) & =\sum_{e=1}^{E} \operatorname{FLDISC}(e, L)
\end{aligned}
$$

The relationship between age $T$ and length $L$ is described by the usual von Bertalanffy equation.

$$
\begin{aligned}
& \mathrm{L}(\mathrm{~T})=\mathrm{L} 8(1-\exp (-\mathrm{K}(\mathrm{~T}-\mathrm{TO}))) \text { or } \\
& \mathrm{T}(\mathrm{~L})=\mathrm{TO}-\mathrm{LOG}(1-\mathrm{L} / \mathrm{L} 8) / \mathrm{K}
\end{aligned}
$$

SL, RL, DL and FL can be concidered as functions of age by

$$
\begin{aligned}
S(e, T) & =\operatorname{SL}(e, L(T)) \\
R(e, T) & =\operatorname{RL}(e, L(T)) \\
D(e, T) & =\operatorname{DL}(e, L(T)) \\
F(e, T) & =\operatorname{FL}(e, L(T)) \\
\operatorname{FLAND}(e, T) & =\operatorname{FLLAND}(e, L(T)) \\
\operatorname{FDISC}(e, T) & =\operatorname{FLDISC}(e, L(T)) \\
F(T) & =\Sigma F(e, T), \operatorname{FLAND}(T)=\sum_{e} \operatorname{FLAND}(e, T) \\
\operatorname{FDISC}(T) & =\sum_{e} \operatorname{FDISC}(e, T)
\end{aligned}
$$

4.2 Population Dynamics.

The number of survivors of the yearclass is determined from

$$
\frac{\mathrm{dN}(T)}{\mathrm{dT}}=-\left(\mathrm{F}^{\prime}(\mathrm{T})+M(T)\right) N(T)
$$

where $M(T)$ is the natural mortality at age $T$.
The number caught is given by

$$
\frac{\mathrm{dC}(\mathrm{~T})}{\mathrm{dT}}=\mathrm{F}(\mathrm{~T}) \mathrm{n}(\mathrm{~T})
$$

where $C(T)$ is the total number caught in the time period from TI to $T$.
The number caught by fleet $e$ is given by

$$
\frac{\mathrm{dC}(e, T)}{\mathrm{dT}}=F(e, T) N(T)
$$

where $C(e, T)$ is the number caught by fishing fleet $e$ in the time period from TI to T .

$$
C(e, T)=\int_{T I}^{T} F(e, t) N(t) d t
$$

The number landed by fleet $e$ is

$$
\operatorname{LAND}(e, T)=\int_{T I}^{T} D(e, t) F(e, t) N(t) d t
$$

and the number discarded is:

$$
\operatorname{DISC}(e, T)=\int_{T I}^{T}(1-D(e, t)) F(e, t) N(t) d t
$$

The total number landed and discarded are

$$
\operatorname{LAND}(T)=\sum_{e} \operatorname{LAND}(e, T) \text { and } \operatorname{DISC}(T)=\sum_{e} \operatorname{DISC}(e, T)
$$

Total number caught, landed and discarded by all fleets are $\Sigma C(e, T), \Sigma L A N D(e, T)$ and $\operatorname{EDISC}(e, T)$ respectively.

The number caught in time period from T 1 to T 2 is:

$$
C(T 2)-C(T 1)=\int_{T 1}^{T 2} F(t) N(t) d t
$$

The number caught of lengths between L1 and L2 is:

$$
C(T(L 2))-C(T(L 1))=\int_{T(L 1)}^{T(L 2)} F(t) N(t) d t
$$

The number caught by fishing fleet $e$ of lengths between L 1 and L 2 is:

$$
C(e, T(L 2))-C(e, T(L 1))=\int_{T(L 1)}^{T(L 2)} F(e, t) N(t) d t
$$

### 4.3 Number caught per length group.

Let the catch be divided into length groups by $L G(1), L G(2), \ldots . ., L G(I)$.
A fish belongs to length group i, if

$$
\mathrm{LG}(\mathrm{i}) \leqq \text { length of the fish }<\mathrm{LG}(i+1)
$$

Let $\quad T G(i)=T(L G(i))$
Then the number caught in length group $i$ is:

$$
C(T G(i+1)-C(T G(i))
$$

and the number caught in length group $i$ by fishing fleet e is:

$$
C(e, T G(i+1))-C(T G(i))
$$

The number landed from length group $i$ by fishing fleet $e$ is:

$$
\begin{equation*}
C L(e, i)=\operatorname{LAND}(e, T G(i+1))-\operatorname{LAND}(e, T G(i)) \tag{1}
\end{equation*}
$$

and the number discarded is:

$$
\begin{equation*}
C D(e, i)=\operatorname{DISC}(e, \operatorname{TG}(i+1))-\operatorname{DISC}(e, T G(i)) \tag{2}
\end{equation*}
$$

The total catch per length group is designated:

$$
\begin{equation*}
C T(e, i)=C L(e, i)+C D(e, i) \tag{3}
\end{equation*}
$$

CT and CL are the basic observations of this analysis.
4.4 Parameters of the selection curves.

The gear selection curve $S L(e, L)$ is defined by

$$
\operatorname{SL}(e, L)=\frac{\operatorname{GSEL}(e, L)}{1+\operatorname{GSEL}(e, L)} \cdot \frac{\operatorname{DGSEL}(e, L)}{1+\operatorname{DGSEL}(e, L)}
$$

(for detailed explanation of the mathematical aspects of this formula, see appendix A)
where:

$$
\operatorname{GSEL}(e, L)=\exp \frac{(\mathrm{L}-\mathrm{L} 50 \%(\mathrm{e})) \log 3}{\mathrm{~L} 75 \%(\mathrm{e})-\mathrm{L} 50 \%(\mathrm{e})}
$$

and

$$
\operatorname{DGSEL}(e, L)=\exp \frac{(L-D L 50 \%(e)) \log 3}{\operatorname{DL} 75 \%(e)-\operatorname{DL50\% (e)}}
$$

$\mathrm{L} 50 \%$ (e) is the length at which $50 \%$ of the fish entering the gear of fleet $e$ are retained by the gear.

L75\% (e) is the length at which $75 \%$ of the fish entering the gear of fleet $e$ are retained by the gear.
$L 50 \%$ and $L 75 \%$ describe the left hand side if the gear selection curve (the ascending part), and DL75\% are the equivalent parameters for the right hand side of the curve (the decending part), as illustrated in Figure 3.


Fig. 3. Gear selection curve

The expression of the recruitment curve is mathematically equivalent to that of the gear selection curve:

$$
\operatorname{RL}(e, L)=\frac{\operatorname{RSEL}(e, L)}{1+\operatorname{RSEL}(e, L)} \cdot \frac{\operatorname{DRESL}(e, L)}{1+\operatorname{DRESL}(e, L)}
$$

where

$$
\begin{aligned}
& \operatorname{RSEL}(e, L)=\exp \left[\frac{(\operatorname{L}-\operatorname{RL} 50 \%(e)) \log 3}{\operatorname{RL} 75 \%(e)-\operatorname{RL} 50 \%(e)}\right] \\
& \operatorname{DRSEL}(e, L)=\exp \left[\frac{(\operatorname{L}-\operatorname{DRL} L 0 \%(e)) \log 3}{\operatorname{DRL} 75 \%(e)-\operatorname{DRL} 50 \%(e)}\right]
\end{aligned}
$$

(see Figure 4 ).


Fig 4. Recruitment (migratịon.) curve

The discard curve, $D L(e, L)$ ( $=$ the fraction not not discarded) does not have a descending part as only small fish are assumed to be discarded, For the ascending part of the curve the expression is equivalent to those of gear selection and recruitment:

$$
\operatorname{DL}(e, L)=\frac{\operatorname{DISEL}(e, L)}{1+\operatorname{DISEL}(e, L)}
$$

where

$$
\operatorname{DISEL}(e, L)=\exp \left[\frac{(\operatorname{L-DIL50\% }(e)) \log 3}{\operatorname{DIL} 75 \%(e)-\operatorname{DIL50\% (e))}]}\right]
$$



Fig. 5. Discard curve. DL is the fraction not discarded and 1- DL is the fraction dicarded of the fish caught.

Thus, landing and discard mortality

$$
\begin{aligned}
& \operatorname{FLLAND}(e, L)=\operatorname{DL}(e, L) \operatorname{SL}(e, L) \operatorname{RL}(e, L) \operatorname{EF}(e) \text { and } \\
& \operatorname{FLDISC}(e, L)=(1-D L(e, L)) \quad \operatorname{SL}(e, L) \operatorname{RL}(e, L) \operatorname{EF}(e)
\end{aligned}
$$

are determined by the set of parameters, one set for each fishery:

| EF |  | total fishing mortality on age ject to maximum exploitation |
| :---: | :---: | :---: |
| L50\% | , $575 \%$ | : ascending gear selection curve |
| DL50\% | , DL75\% | : descending gear selection curve |
| RL50\% | , RL50\% | : ascending recruitment curve |
| DRL50\% | , DRL75\% | : decending recruitment curve |
| DIL50\% | , DIL75\% | : discard curve |

The parameter to be estimated is $\operatorname{MESH}(\mathrm{e})$, the effective mesh size of fishing fleet e.

MESH(e) is determined by $L 50 \%(e)$ and the
selection factor SEL(e)

$$
\begin{equation*}
L 50 \%(e)=\operatorname{MESH}(e) \operatorname{SEL}(e) \tag{4}
\end{equation*}
$$

In the present analysis all parameters, except for MESH(e) (and consequently L50\%(e)) are assumed to be known from independent investigations. Instead of assuming $\mathrm{L} 75 \%(e)$ to be known, the ratio

$$
\mathrm{FAC}(\mathrm{e})=\frac{\mathrm{L} 75 \%(\mathrm{e})}{\mathrm{L} 50 \%(\mathrm{e})}
$$

is assumed to be known, so that the estimation of $\mathrm{L} 75 \%$ follows immedately from the estimate of MESH (or L50\%).

For some fisheries derecruitment does not occur, or fishes are not discarded what ever the size of the fishes.

In the case of no derecruitment this is simulated in the program by setting the lengths defining the derecruitment DRL50\% and DRL75\% to some suitable values well above the length range simulated.

If fishes are not discarded at all, this is simulated by setting DIL50\% to some suitable value. (=0)

### 4.5 Observed and theoretical length distribution of catches.

Equations (1), (2) and (3) define the theoretical length distributions of catches as predicted by the model.

Let us rewrite the expressions of landings and discards at length (Eqs. (1) and (2)) by inserting the symbols for selection curves:
$C L(e, i)=$
TG(i+1)
$\int \operatorname{FLLAND}(e, L(t)) N(t) d t=$ TG(i)

TG(i+1)
$\int_{T G(i)} D L(e, L(t)) S L(e, L(t)) R L(e, L(t)) E F(e) N(t) d t$
and

$$
\begin{aligned}
& C D(e, i)=\int_{T G(i)}^{T G(i+1)} \operatorname{FLDISC}(e, L(t)) N(t) d t= \\
& \int_{T G(i)}^{T G(i+1)}(1-D L(e, L(t)) \operatorname{SL}(e, L(t)) \operatorname{RL}(e, L(t)) \operatorname{EF}(e) N(t) d t
\end{aligned}
$$

The estimates of numbers landed by length (age groups) are based on samples from commercial landings.

Usually this observed length distribution of landings are taken as the average of a number of years in which the gears are assumed to have remained unchanged. The observations are designated:

$$
\begin{aligned}
\operatorname{OBSCL}(e, i)= & \text { observed number of fish landed in length group i } \\
& \text { by fishing fleet e. }
\end{aligned}
$$

The estimation problem is to find values of $\operatorname{MESH}(e)$, $e=1,2, \ldots, E$ so that

$$
\begin{equation*}
\mathrm{e}=1 \sum_{\mathrm{i}}^{E}(\operatorname{CL}(\mathrm{e}, \mathrm{i})-\operatorname{OBSCL}(\mathrm{e}, \mathrm{i}))^{2} \tag{6}
\end{equation*}
$$

is minimized.
To spell out the relations between the various parameters and variables (6) may be rewritten by incerting Eq (4) into Eq (5) and Eq (5) in to Eq (6) and L75\% $=\mathrm{L} 50 \%=$ (FAC - 1) L50\%.

$$
E \quad T G(i+1)
$$

$\sum \sum_{i} \int D L(e, L(t)) R L(e, L(t)) E F(e)$.
$e=1$ i TG(i)
$\frac{\exp \left[\frac{(L(t)-\operatorname{MESH}(e) \cdot \operatorname{SEL}(e)) \log (3))}{(\operatorname{FAC}(e)-1) \operatorname{MESH}(e) \operatorname{SEL}(e)}\right]}{\left.N(t) d t-\operatorname{OBSCL}(e, i)^{2}\right)}$
$1+\exp \left[\frac{\mathrm{L}(\mathrm{t})-\operatorname{MESH}(\mathrm{e}) \operatorname{SEL}(\mathrm{e}) \log (3)}{(\operatorname{FAC}(\mathrm{e})-1) \operatorname{MESH}(\mathrm{e}) \operatorname{SEL}(\mathrm{e})}\right]$
Thus, the problem is to determine the $\operatorname{MESH}(e)$ so that the expression in Eq(7) is minimized, when all other terms of $\mathrm{Eq}(7)$ are known parameters or observations.

The values of $\operatorname{MESH}(\mathrm{e})$, $e=1,2, \ldots, E$ which minizes the sum of squares of deviations Eq(7) are the effective mesh sizes.

With some appropriate changes in integration limits in Eq(7) an substituting the observed length distribution with an observed age distributions can be used in the same way. The summation in eq. 7 is then done over agegroups rather than length groups.

## Estimation procedure.

To determine the value of the sums of squares Eq(7) requires the solution of a set of simultaneous differential equations. The differential equations are those which describe the dynamies of population and landings.
$\frac{d N(t)}{d t}=-(M(t)+F(t)) N(t)$
$\frac{\operatorname{dLAND}(1, t)}{d t}=D(1, t) F L(1, t) N(t)$
$\frac{\operatorname{dLAND}(2, t)}{d t}=D(2, t) \operatorname{FL}(2, t) N(t)$
$\frac{\operatorname{dLAND}(E, t)}{d t}=D(E, t) F L(E, t) N(t)$

To determine a unique solution of Eqs (8) a intitial values of the variables $N$ and LAND are required.

The initial value of LAND is obviously zero.

$$
\operatorname{LAND}(e, T I)=0 \text { for all } e .
$$

The initial value of N is arbitrarily assigned the value 1000:

$$
N(T I)=1000
$$

That means that all calculations are made on a relative basis.
To make the observations comparable to the theoretical, OBSCL should be expressed in relative terms. This could be done as follows:

The observed relative length distribution of catches is defined

$$
\frac{\operatorname{OBSCL}(e, i)}{\sum_{e} \sum_{i} \operatorname{OBSCL}(e, i)}=\operatorname{ROBSCL}(e, i)
$$

'l'he estimated (theoretical) distribution is defined

$$
\frac{C L(e, i)}{\sum_{e} \sum_{i} C L(e, i)}=\operatorname{RCL}(e, i)
$$

The sum of squares to be minimized (Eq. (6)) becomes

$$
\sum_{e} \sum_{i}[\operatorname{RCL}(e, i)-\operatorname{ROBSCL}(e, i\rangle]^{2}
$$

This object function considers the observations relative to the total catch.
Another possibility is to consider the catches of each fleet relative to the catch of the fleet, i.e. to define relative observations as

$$
\frac{\operatorname{ROBSCL}(e, i)}{\sum_{i}^{\operatorname{ROBSCL}(e, i)}}
$$

and relative estimates as

$$
\frac{\operatorname{RCL}(e, i)}{\sum_{I} \operatorname{RCL}(e, i)}
$$

The latter approach considerseach fleet as being of equal importance, whereas the first approach considers those fleets with the largest catch as the most important ones.
To make these to possibilities optional for the users, the object function has been written in the general form
$\sum_{e} \sum_{i}(\operatorname{RCL}(e, i)-\operatorname{ROBSCL}(e, i))^{2} \operatorname{COEF}(e)$
where the coefficients, COEF, can be chosen by the user.
E.g. If $\operatorname{CoEF}(e)=1$ for all e, catches are relative to total catch. If we put $\left.\operatorname{COEF}(e)=1 / \sum_{i} \operatorname{RCL}(e, i)\right)^{2}$
the object function will approximately be as if length distibutions were relative to catch of each fleet separately.

Having solved Eqs (8) the sums of squares and the number discarded (Eq(5)) can be found.

The differential equations can be solved by some numerical solution (E.g. RungeKutta, see e.g. Ralston, 1956).

In App. A a numerical example of (8) is discussed.
To minimize the sum of squares of deviations (Eq (7)) some numerical method must be used.

Two optimization routines have been tested. One approach was the Nelder and Mead algorithm, which is a relative simple general purpose minimization algorithm. The other approach was the algorithm VAOSA of the Harwell Subroutine Library, which is a somewhat more sophisticated algorithm designed especially to minimize a sum of squares.

However, from a biological point of view these computational technical details are of a limited interest, and only a brief description of the procedure is given here.

The optimization algorithm works as an iterative process. That, is the algorithm should be provided with an initial guess on the unknown variables (the MESHs), and based on that the algorithm calculates an improved estimate. This process continues until the estimate in the current and in the foregoing iteration are approximately equal.

The effect of changing initial guesses of mesh sizes is discussed in App. A, which also discusses some other aspects of the robustness of the algorithm VAOSA.

## 5. FORMAL DESCRIPTION DF STEP 2

The effect of changing gear parameters can be assessed by running the system (8) in the forecast mode.

This approach is somewhat different from that one usually applied by ICES WGs when making forecast calculations.

The commonly used approach is to continue the VPA-calculations into future years, i.e. the system is described by a time discrete model.

The estimates of effective mesh sizes can easily be imcorporated in the usual catch prediction procedure, by converting the selection curves as functions of lengths into functions of age. Such a conversion is illustrated in Figure 5 .


Fig. 6 . Converting a continous selection curve into a time discrete selection curve.

The assessment of mesh size may be done within the scope of a single species assessment method, but it may also be carried out in more complicated models. In Sparre, (1980), a detailed description of mesh assessments in the commonly used time discrete model is given.

This work also describes how mesh assessment can be incorporated into a species interaction and technical interaction model.

As the technical interaction is considered an important factor in the assessment the simple approach of running the system of Eqs. (8) should be treated with a certain reservation.

For a discussion of these aspects, see Sparre, 1980.
5.1. Formal description of the time continous single species prognosis model.

As the time continous single species model has been used in some assessments of mesh changes (Anon, 1974, 1977, 1979 and 1980) a formal description of the method is given.

Let $\operatorname{OMESH}(\mathrm{e})$ designate the old mesh size and $\operatorname{NMESH}(e)$ the new mesh size. Let the change of mesh sizes occur at the end of year T1 (see Figure 6 ;page 22)

In this approach, it is assumed that the population is in a steady state situation before time T1. After the change of mesh sizes the parameters of the system are assumed to remain constant, which implies that the system ends up in a new steady state after a certain transient period.

This approach should only be applied as a strategic model, i.e. to assess the long term effect of a mesh change.

The output of the model contains a description of the transient period between the old steady state and the new steady state (see Figure 6 ), but these results should be treated with a certain reservation, as the assumption of a stable situation is not fulfilled.

In the following it should be assumed that we are in a constant parameter system. (For a discussion of this assumption, see Sparre, 1980).

Let $T 2$ designate the time at which the transient period is over (see Figure $\delta$ ).
In the present context we consider all yearclasses during one year, and not as in the foregoing sections, a yearclass during its life span.

To describe such a system the notation must be modified. Let $N(y, t)$ designate the number of survivors from yearclass $y$ at age $t$. Thus at the beginning of year T1 the stock is composed of the following yearclasses.

$$
N(T 1-1,0), N(T 1-2,1), N(T 1-3,2), \ldots .
$$

and at time $u$ in year $\mathrm{T} 1(0 \leqq u \leqq 1)$ the stock is composed of

$$
N(T 1-1, u), N(T 1-2,1+u), N(T 1-3,2+u), \ldots \ldots
$$

Let T3 be some year after the end of the transient period.
The number landed in agegroups a before change of mesh size (e.g. in year T1) by fleet e is

$$
\operatorname{LANDY}(e, T 1, a)=\int_{0}^{1} \operatorname{OFLAND}(e, a+u) N(T 1-a-1, a+u) d u
$$

where OFLAND is the landing mortality defined by the old gear selection curve (and discard and recruitment curve).
i.e.

$$
\operatorname{OFLAND}(e, a+u)=D(e, a+u) R(e, a+u) \operatorname{EF}(e) \cdot \frac{\exp \left[\frac{L(a+u)-\operatorname{OMESH}(e) \operatorname{SEL}(e) \log 3}{(\operatorname{FAC}(e)-1) \operatorname{OMESH}(e) \operatorname{SEL}(e)}\right]}{1+\exp \left[\frac{L(a+u)-\operatorname{OMESH}(e) \operatorname{SEL}(e) \log 3}{(\operatorname{FAC}(e)-1) \operatorname{OMESH}(e) \operatorname{SEL}(e)}\right]}
$$

After the transient period the landings in, say, year T3 become.

$$
\operatorname{LANDY}(e, T 3, a)=\int_{0}^{1} \operatorname{NFLAND}(e, a+u) N(T 3-a+1, a+u) d u
$$

where NFLAND is defined by the new gear selection curve.
The landings are determined by solving the system of differential equations (Eqs (8)) for both the new and the old parameters.

Discards are given by

$$
\operatorname{DISCY}(e, T 1, a)=\int_{0}^{1} O F D \operatorname{ISC}(e, a+u) N(T 1-a-1, a+u) d u
$$

and

$$
\operatorname{DISCY}(e, T 3, a)=\int_{0}^{1} \operatorname{NFDISC}(e, a+u) N(T 3-a-1, a+u) d u
$$

and as the landings they are found by solving a system of differential equations:

$$
\begin{aligned}
\frac{\operatorname{dDISC}(e, t)}{d t}= & (1-D(e, t)) F(e, t) N(t) \\
& e=1,2, \ldots \ldots, E
\end{aligned}
$$

By the above described prodedure the landings and discards before and after the transient period are determined.

Let $W(a)$ be the average body weight of an a year old fish.
The yield of fleet e before the transient period is

$$
\sum_{a} \operatorname{LANDY}(e, T 1, a) W(a)=\operatorname{YIELD}(e, T 1)
$$

and the yield after the transient period is

$$
\sum_{a} \operatorname{LANDY}(e, T 3, a) W(a)=Y \operatorname{IELD}(e, T 3)
$$

Discards are found by similar expressions.
The yields in the transient period may be assessed in the following way:
Let T 4 be some year in the transient period ( $\mathrm{T} 1<\mathrm{T} 4<\mathrm{T} 2$ ).
Let $\mathrm{A} 4=\mathrm{T} 4-\mathrm{T} 1$, then be

$$
\operatorname{YIELD}(e, T 4)=
$$

$\sum_{a}^{A 4} \operatorname{LANDY}(e, T 3, a) W(a)+$
$a=1$

$$
\sum_{a=A 4+1} \frac{N(T 1, a)}{N(T 3, a)} \operatorname{LAND}(e, T 3, a) W(a)
$$

Fig 6. Time table of mesh change assessment.

CHANGE OF GEARS


| a | : index of age group. |
| :---: | :---: |
| $C(e, T)$ | : the number caught in the time period $T$ I to $T$ by fleet $e$. |
| $C(T)$ | : $\sum_{\mathrm{e}} C(e, T)$ total number caught in the time period from TI to $T$ by all fleets. |
| $C D(e, i)$ | : the number from length group i discarded by fleet e. |
| CL (e,i) | : the number landed of length group i by fleet e, theoretical value. |
| CT(e,i) | : total number caught (landings + discards) of length group i by fleet e. |
| $D(e, T)$ | discard curve (the fraction not discarded) as a function of age, for fleet e. |
| DL (e, L ) | discard curve (the fraction not discarded) as a function of length, for fleet e. |
| DL50\% (e) | : 50 percent gear selection length of fleet e, for the descending part of the curve. |
| DL75\% (e) | : 75 percent gear selection length of fleet $e$, for the descending part of the curve. |
| DRL 50\% (e) | : 50 percent recruitment length of fleet $e$, for the descending part of the curve. |
| DRL75\% (e) | : 75 percent recruitment length of fleet $e$, for the descending part of the curve. |
| DIL50\% (e) | : 50 percent discard length of fleet e. |
| DIL75\% (e) | : 75 percent discard length of fleet e. |
| DGSEL (e, L) | : term in the descending factor of the gear selection curve of fleet e. |
| $\operatorname{DISEL}(\mathrm{e}, \mathrm{L})$ | : term in the discard curve of fleet e (the fraction not discarded). |
| $\operatorname{DRSEL}(\mathrm{e}, \mathrm{L})$ | : term in the descending factor of the recruitment curve of fleet e. |
| DISC(e, T ) | : the number discarded in the time period from $T I$ to $T$ by fleet e. |
| E | : number of fleets |
| e | : index of fleet |
| $E F(e)$ | : fishing mortality exerted by fleet e, on age groups subject to maximum exploitation. |
| $F(e, T)$ | : fishing mortality at age T exerted by fleet e . |
| $F(\mathrm{~T})$ | : $\sum_{e} F(e, T)$ : total fishing mortality at age T . |
| $F L(e, L)$ | : fishing mortality at length L exerted by fleet e. |
| FL(L) | : $\sum_{e} F L(e, L)$ total fishing mortality at length L. |
| FLAND (e, T) | : landing mortality at age T exerted by fleet e. |
| FDISC(e, T) | : discard mortality at age $T$ exerted by fleet e. |
| FLAND ( T ) | : $\sum_{e} \operatorname{FLAND}(e, T)$ total landing mortality at age $T$. |
| FDISC(T) | : $\sum_{e} \operatorname{FDISC}(e, T)$ total discard mortality at age T . |
| FLLAND (e, L) | : landing mortality at length L , exerted by fleet e. |
| $\operatorname{FLDISC}(e, L)$ | : discard mortality at length L, exerted by fleet e. |


| FLLAND (L) | : $\sum_{e} \operatorname{FLLAND}(\mathrm{e}, \mathrm{L})$ total landing mortality at length L |
| :---: | :---: |
| ELDISC(L) | : $\sum_{e} \operatorname{FLDISC}(e, L)$ total discard mortality at length L. |
| FAC(e) | : L75\% (e)/L50\% (e) |
| GSEL (e, L ) | : term in the ascending factor of the gear selection curve of fleet e. |
| i | : index of length group |
| K | : von Bertalanffy growth parameter. |
| L | : length |
| $L(t)$ | : length at age $t$ (the von Bertalanffy growth equation: L8(1-exp ( $-K(t-T O))$ ). ) |
| L8 | : assymptotic length in the von Bertalanffy equation. |
| LG(i) | : length group i. Length group $i$ is defined as the interval between $L G(i)$ and $L G(i+1)$. |
| $\operatorname{LAND}(\mathrm{e}, \mathrm{T})$ | : number landed in the time period from TI to T by fleet e. |
| L50\% (e) | : 50 percent gear selection length of fleet e for the ascending part of the curve. |
| L75\% (e) | : 75 percent gear selection length of fleet $e$, for the ascending part of the curve. |
| $\operatorname{LANDY}(\mathrm{e}, \mathrm{T}, \mathrm{a})$ | : landings of agegroup a in gear $T$ by fleet $e$. (only used in the prognosis part of the model). |
| M ( T ) | : natural mortality at age T. |
| $N(T)$ | : stock number at age T . |
| $N(T, a)$ | : the stock number at age a in year $T$ (only used in the prognosis part of the model). |
| NMESH (e) | : "new mesh size", meshsize after change of gear of fleet e. |
| NFLAND (e, a) | : "new" landing mortality (after change of gear) of fleet e, on agegroup a. |
| NFDISC(e,a) | : "new" discard mortality (after change of gear) of fleet e, on agegroup a. |
| $\operatorname{OBSCL}(e, i)$ | : observed number landed of length group i fish by fleet e. |
| OMESH (e) | : "old mesh size" before change of gear of fleet e. |
| $\operatorname{OFLAND}(\mathrm{e}, \mathrm{a})$ | : "old" landing mortality (before change of gear) of fleet e, on agegroup a. |
| $\operatorname{OFDISC}(\mathrm{e}, \mathrm{a})$ | : "old" discard mortality (before change of gear) of fleet e, on agegroup a. |
| $R(e, T)$ | : recruitment curve for fleet e, as a function of age. |
| RL(e, L ) | : recruitment curve for fleet e, as a function of length. |
| $\operatorname{RCL}(\mathrm{e}, \mathrm{i})$ | : relative number of length group i landed by fleet e. Estimated (theoretical) value. |
| $\operatorname{ROBSCL}(e, i)$ | : relative number of length group i landed by fleet e, observed value. |


| $\operatorname{RSEL}(\mathrm{e}, \mathrm{L})$ | : term in the ascending factor of the recruitment curve. |
| :---: | :---: |
| RL50\% (e) | : $50 \%$ recruitment length of fleet $e$, for the ascending part of the curve. |
| RL75\% (e) | $75 \%$ recruitment length of fleet $e$, for the ascending part of the curve. |
| $S(e, T)$ | : gear selection curve as a function of age, for fleet e. |
| SLI (e, L ) | : gear selection curve as a function of length, for fleet e. |
| SEL (e) | : selection factor of fleet e. |
| T | : time (year). |
| TI | : youngest age considered. |
| TO | : von Bertalanffy growth parameter. |
| T (L) | : the inverse von Bertalanffy function. |
| TG(i) | : age corresponding to length LG(i). |
| T1 | : year when years are changed. |
| T2 | : last year of transient period. |
| T3 | : some year after the transient period. |
| T4 | : some year in the transient period. |
| W(a) | : average body weight of agegroup a. |
| YIELD ( $\mathrm{e}, \mathrm{T}$ ) | : yield of fleet $e$ in year $T$. |

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Table 1. Sensitivity analysis of the best estimates of effective mesh size estimates for North East Arctic Cod (CM 1979 (G:20). Length and age distributions 1967 - 1977.

| $10 \%$ change of | Length distibutions | Age distibutions |
| :--- | :--- | :--- |
| parameters | $\%$ change of average | $\%$ change in average |
|  | mesh size est. | mesh size est. |

Growth parameters:

| L8 | -3 | +15 |
| :--- | ---: | ---: |
| K | -3 | +13 |
| T0 | 0 | -1 |

Mortalities:

| $\sum \operatorname{EF}(\mathrm{e})^{1)}(\mathrm{M}=0.1)$ | - 3 | - | 3 |
| :---: | :---: | :---: | :---: |
| $\sum_{e} \operatorname{EF}(\mathrm{e})^{2}$ ( $\left.M=0.2\right)$ | + 2 |  | 2 |
| M 1) ( $\left.\sum \operatorname{EFF}(\mathrm{e})=0.98\right)$ | + 2 |  | 2 |
| $\left.\left.M^{\text {® }}\right) \quad \mathrm{e} E F(e)=1.0\right)$ | + 1 | + | 1 |

Selection curve.
$\mathrm{L} \% 5 \% / \mathrm{L} 50 \% 4$ ) $+\quad-1$

Recruitment. - 3

- 4

Derecruitment. 5) +
Discarding. -12 - 3

1) With the restriction $\sum_{e} E F(e)+M=1.2$
2) $M$ is unchanged
3) $\sum_{e} E F(e)$ is unchanged.
4) The $10 \%$ increase is calculated as (L75\%/L50\%-1) $1.1+\mathbb{L} 75 \% / L 50 \%$
5. The $10 \%$ increase in the derecruitment parameters is only done for those 5 fisheries that have a derecruitment curve within the simulated range $10-135 \mathrm{~cm}$. The corresponding change of the estimates effective mesh size is the average for the same 5 fisheries.

## APPENDIX A.

## The effect of stochastic variation of catch observations and initial quess ON EFFECTIVE MESH SIZES.

1.INTRODUCTION.

The iterative procedure applied to estimate the effective mesh size, requires an initial guess on the mesh sizes, to start the process. The effect on the final estimates of mesh size caused by changing the initial guesses, is assessed in this appendix.

The catch at length observation may be considered as the sum of a "true value" plus a stochastic term. The effect on estimates of mesh sizes caused by the stochastic term is assessed.

The exercises are based on a hypothetical stock and four hypotetical fleets. "True catches" are constructed so that all assumptions of the model are fulfilled. Thus, in this exercise we are in the favourable position to know the "true values" of parameters. The effects of varying input data by stochastic simulation is assessed by comparing the simulated estimates of mesh size with the "true mesh sizes".

## 2. CONSTRUCTION OF THE HYPOTHETICAL STOCK AND THE TRUE CATCHES.

Growth parameter are chosen to be:

$$
\mathrm{L} 8=131 \mathrm{~cm} \quad \mathrm{~K}=.13 \text { and } \mathrm{TO}=0 \text {; }
$$

Natural mortality is assumed to remain constant for all ages

$$
M=0.2
$$

The number of fleets is: $E=4$.

Recruitment -, discard and right hand side of gear selection curves are put equal to 1.0 for all lengths. I.e. all length groups are assumed to be fully recruited, no discarding is assumed to occur, and no decending slope on the gear selection curve is assumed.

For all fishing fleets it is assumed that

$$
\begin{aligned}
\mathrm{FAC}(\mathrm{e}) & =1.1 \text { or } \\
1.1 \mathrm{~L} 50 \%(\mathrm{e}) & =\mathrm{L} 75 \%(\mathrm{e})
\end{aligned}
$$

when simulating catch distribution by solving the differential Eqs. (8), the following parameters were used:

| Fishing fleet <br> $e$ | L50\%(e) | L75\%(e) | SEL(e) | EF (e) | "true" MESH (e ) <br> cm |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 30.0 | 33.0 | 3.0 | .2 | 10.0 |
| 2 | 45.0 | 49.5 | 3.0 | .1 | 15.0 |
| 3 | 60.0 | 66.0 | 3.0 | .3 | 20.0 |
| 4 | 45.0 | 49.5 | 3.0 | .3 | 15.0 |

Table A1.

The term $(\log 3) /(L 75 \%(e)-L 50 \%(e))$ in the expression for gear selection becomes $.366, .244, .184$ and .244 for the four fleets resp.

Inserting the parameters into Eqs. (8) the set of differential equations becomes:
$I: \quad \frac{d N(t)}{d t}=-\left(.2+\frac{\exp (.366(L(t)-30.0))}{1+\exp (.366(L(t))-30.0))} .2+\frac{\exp (.244(L(t)-45.0))}{1+\exp (.244(L(t)-45.0))} .1\right.$
$\left.+\frac{\exp (.183(L(t)-60.0))}{1+\exp (.183(L(t)-60.0))} \cdot 3+\frac{\exp (.244(L(t)-45.0))}{1+\exp (.244(L(t)-45.0))} \cdot 3\right) N(t)$
II: $\quad \frac{\operatorname{dLAND}(1, t)}{d t}=\frac{\exp (.366(L(t)-30.0))}{1+\exp (.366(L(t)-30.0))} \cdot 2 N(t)$
III: $\quad \frac{\operatorname{dLAND}(2, t)}{d t}=\frac{\exp (.366(L(t)-30.0))}{1+\exp (.244(L(t)-45.0))} \cdot 1 N(t)$
IV: $\quad \frac{\operatorname{dLAND}(3, t)}{d t}=\frac{\exp (.183(L(t)-60.0))}{1+\exp (.183(L(t)-60.0))} \cdot 3 N(t)$
$V: \quad \frac{\operatorname{dLAND}(4, t)}{\operatorname{dt}}=\frac{\exp (.244(L(t)-45.0))}{1+\exp (.244(\mathrm{~L}(\mathrm{t})-45.0))} \cdot 3 N(t)$
where $L(t)=131.0(1-\exp (-.13 t))$

The catches are divided into 5 cm length groups, altogether 22 length groups.
The solution of the system I-V, then yields the "true" catches. These are shown in Figure A1 by dotted lines.
3. TESTING THE EFFECT OF CHANGING INITIAL GUESS ON MESH SIZE.

A number of testruns with different initial guesses on mesh sizes were performed. The simulated observations were those defined be the solutions of Eqs. I-V (the dotted lines on Figure A1). I.e. the observations to be expected if there were no discrepancy between the model and the real world.

It turned out that except for those cases where the initial guesses were given extreme values the algorithm was able to find the exact correct values. When the algorithm failed, the results were vary far from the correct ones.

To illustrate the results, five examples of initial guesses are shown in the table below.

| Fishing fleet | "true" mesh <br> size |  | initial guess on mesh sizes |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |

Thus of the initial guesses were given extremely low values, the algorithm failed to find the correct solutions.

However, the initial guesses which resulted in failures, are so extreme, that it is not likely that they would be chosen by a person with a minimum knowledge of the fishery.

The conclusion from this exercise is that the routine VA05A is rather robust to changes of the initial guesses.

## 4. TESTING THE EFFECT OF STOCHASTIC VARIATION OF THE OBSERVATIONS.

With the aid of a random number generator all observations were made stochastic variables. Let TRUECL(e,i) designate the "true values" of catch distribution on length groups. I.e. TRUECL(e,i) is the solution of Eqs. I-V and they are shown in Figure A1 by dotted lines. The simulated stochastic observations applied in the exercise are given by

$$
\operatorname{OBSCL}(e, i)=\operatorname{TRUECL}(e, i)(1+(S T O C H A S T I C ~ T E R M))
$$

The stochastic term are assumed normally distributed with mean value and standard deviation SIGMA.

In Table A2 an example of stochastic simulation is given.

| fishing fleet no. | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: |
| initial guess on <br> mesh size | 5.0 | 5.0 | 5.0 | 5.0 |
| true value of <br> mesh size | 10.0 | 15.0 | 20.0 | 15.0 |
| estimated <br> mesh size | 9.1 | 15.2 | 20.8 | 15.0 |

Table A2
In this simulation SIGMA is given the value o. 40 i.e. the coefficients of variation of each observation (OBSCL) is 40 percent. The simulated length distribution are shown in Figure A1 by full lines.

A large number of stochastic simulations have been made and as indicated by this example the method are relatively insensitive to stochastic variations in the observations.

## 5. EFFECT OF ERRORS IN CATCH NUMBERS ON ESTIMATES OF EFFECTIVE MESH SIZES.

This exercise deals with the functional relationship between the coefficients of variation of the simulated number caught in each length group and the coefficients of variation of the estimates of mesh sizes. The coeff. of var.:
$\operatorname{VAR}(\operatorname{MESH}(e) / \operatorname{MESH}(e)$ was determined by simulation techniques. 50 simulations as described in the preceding section, were performed for each of the four values of SIGMA
$(=\operatorname{OBSCL}(e, i)) 5 \%, 10 \%, 20 \%$ and $40 \%$. For each value of SIGMA the coeff. of var. of MESH estimated from the 50 simulated catch data were calculated.

The same procedure was repeated with a new set of "true mesh sizes".
The results are shown in Figure A2 Figure A2 shows that the coefficient of variance of mesh sizes is about $20 \%$ the value of that for the catches.


Relative landings


Fig. A1 Stochastic simulation of catch-at-length observations dotted line "true"values , full line simulated values.

Relative landings


Relative landings


Figure Al continued.


Fig A2 Relation ship between SIGMA (coeff. of variance of simulated catches by length groups) and the average coefficient of variation of estimate of mesh sizes.

- : true mesh sizes: $10 \mathrm{~cm}, 15 \mathrm{~cm}, 20 \mathrm{~cm}$ and 10 cm resp.

0 : true mesh sizes: $10 \mathrm{~cm}, 11 \mathrm{~cm}, 12 \mathrm{~cm}$ and 13 cm resp.

## APPENDIX B.

## The mathematical expression of a selection curve.

As a mathematical model of gear selection we are looking for a sigmoid shaped curve. The curve should e.g. reflect the probability that a fish entering a trawl is retained by the meshes as a function of fish length. Fig B1 shows such a curve.

$\mathrm{L} 50 \%$ is the length of fish at which $50 \%$ of the fish entering the gear are retained and $L 75 \%$ is the length at which $75 \%$ of the fish are retained. $\mathrm{L} 50 \%$ and $\mathrm{L} 75 \%$ are species and gear specific parameters.

Tanh(L) is a standard mathematical function with a sigmoid shaped graph. (Fig. B2)


Fig. B2

To"move"the tanh-curve to the appropriate place in the coordinate system and to get the right scale tanh should be multiplied by 0.5 and 0.5 should be added and $L 50 \%$ should be subtacted from the independent variable. Teh resulting expression becomes:

$$
\begin{equation*}
0.5+0.5 \tanh (\mathrm{~L}-\mathrm{L} 50 \%) \tag{B1}
\end{equation*}
$$

The graph of function (B1) is given in fig. B3.


To vary the steepness of the curve a new parameter alfa is introduced and the function then becomes

$$
\begin{equation*}
0.5+0.5 \tanh (\mathrm{alfa}(\mathrm{~L}-\mathrm{L} 50 \%)) \tag{B2}
\end{equation*}
$$

where alfa should be given a value so that $0.5 .+0.5 \tanh (a l f a(L 75 \%-L 50 \%))=0.75$. Inserting the definition of $\tanh (\tanh (x)=(\exp (x)-(\exp (-x)) /(\exp (x)+\exp (-x))$ we get that $1 / 2+1 / 2 \tanh (L)=\exp (2 L) /(1+\exp (2 L))$ from which we get

$$
\begin{equation*}
\frac{\exp (2 \text { alfa(L75\%-L50\%)) }}{1+\exp (2 \text { alfa(L75\%-L50\%)) }}=0.75 \tag{B3}
\end{equation*}
$$

Solving this equation with respect to alfa we get

$$
\text { alfa }=\ln (3) /(L 75 \%-L 50 \%)
$$

Writing eq. (B2) and (B3) and inserting the expression for alfa we get

$$
\begin{equation*}
\frac{\exp \left(\frac{\mathrm{L}-\mathrm{L} 50 \%}{\mathrm{~L} 75 \%-\mathrm{L} 50 \%} \ln (3)\right)}{1+\exp \left(\frac{\mathrm{L}-\mathrm{L} 50 \%}{\mathrm{~L} 75 \%-\mathrm{L} 50 \%} \ln (3)\right)} \tag{B4}
\end{equation*}
$$

The last function (B4) has a graph of the shape we need.
Othe mathematical expressions could have been used, and the reason why this special formula is chosen is simply that exp is a standard function on all computers.

