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Estimators in acoustic surveying of fish populations

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Introduction

Estimating stock strength of fish from acoustic survey data with control on the magnitude of possible error of estimate is only possible if the quantities which one wants to estimate is expressed in terms of the parameters of a good mathematical/probabilistic model describing every part of the survey method as well as the relation between fish distribution and received echo signal.

A total model for a survey method using echo-integrator and trawl sample data are proposed and both mathematical and biological/physical problems are discussed.

The author has only some experience from the Norwegian survey method on capelin (see Nakken and Dommasnes [7]), so some ideas and assumptions underlying the model are motivated by things learned on cruises in the Barents Sea.

The results in this work are not sufficient to establish a routine
working scheme at sea which can produce the desirable estimates
together with estimates of their possible errors. However, further
research, mainly on mathematical/probabilistic modelling of the
echo function (see (7)) generated by different fish populations may
bring us nearer this goal. The proceedings [11] gives a good general
view of the present research on acoustic survey problems.

Some relations of acoustics and the echo integration system.

Consider a 3 - dimensional coordinate system with the x, y plane parallel to the sea surface at the same depth as the transducer. The transducer is at point (x, y) moving in a direction making angle δ with the x - axis. See fig. 1.

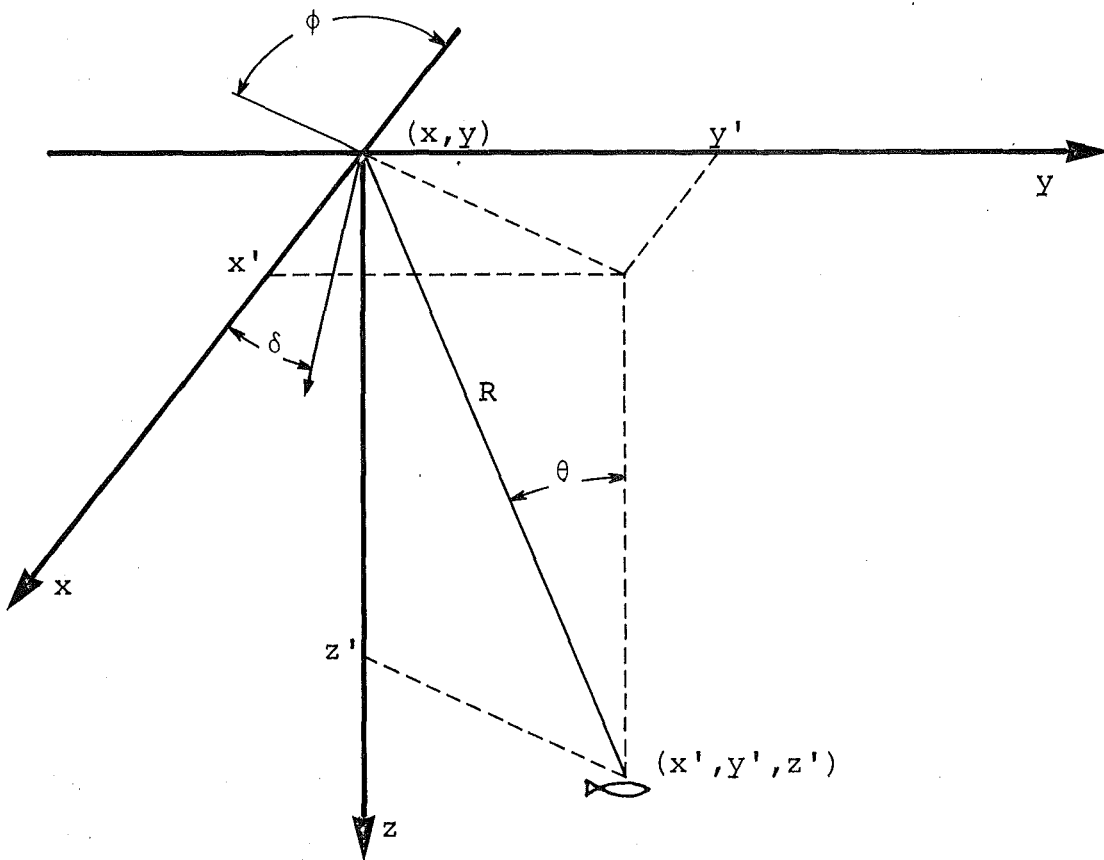A fish is at point x' y' z' with spherical coordinates (R, θ, ϕ)

Fig. 1

After a sound pulse is transmitted, the transducer picks up a part ℓ of the echo-energy reflected by the fish.

Provided the fish is the only reflecting object in the sound cone we call

$$E(x, y) = k \, \ell \, TVG(R) \tag{1}$$

the echo function of the fish. TVG is the time varied gain function, and k is a constant to be defined later. See (17).

Actually the energy picked up is given by

$$\ell = K \frac{e^{-2\varepsilon R}}{R^4} b^2(\theta, \phi-\delta) \frac{\sigma(\tau,\rho)}{4\pi} \tag{2}$$

where

K   is an instrumentation constant

ε is the attenuation coefficient

b   is the directivity pattern function and

$\sigma(\tau,\rho)$ is the scattering cross section of the fish when τ and ρ is its tilt and roll angle respectively relative to the direction transducer fish.   For observations of $\sigma(\tau,\rho)$ for certain fishes see Nakken and Olsen [8]

An important quantity is the integral

$$\iint E(x,y) \, dx \, dy \tag{3}$$

which obviously is independent of $(x' \; y')$.   The integral is over the area where $E(x,y) > 0$.

It can be shown that if $\sigma(\tau,\rho)$ is independent of z', (3) is independent of z' (the depth of the fish)   when

$$TVG(R) = R^2 e^{2\varepsilon R} \tag{4}$$

Let α, β and γ be the tilt, roll and orientation angle respectively of the fish relative to the cartecian coordinatsystem.

When   TVG   is given by (4), we call (3) the echo value of the fish and denote it $p(\alpha,\beta,\gamma,\delta)$.

Let $\sigma'(\alpha,\beta,\gamma,\theta,\phi)$ $= \sigma(\tau,\rho)$ when τ and ρ is expressed in terms of $\alpha,\beta,\gamma,\theta$ and ϕ.

Using (1) (2)   and   (4) and making a change of variables in (3) from (x,y,z) to (R,θ,ϕ), the following expression can be derived.

$$p(\alpha,\beta,\gamma,\delta) = \int_0^{\frac{\pi}{2}} h(\alpha,\beta,\gamma,\delta;\theta) \, tg\theta \, d\theta \tag{5}$$

where $\quad h(\alpha,\beta,\gamma,\delta;\theta) = \int b^2(\theta,\phi-\delta) \, \sigma'(\alpha,\beta,\gamma,\theta,\phi) \, d\theta$

The moving or reference direction δ of the transducer is constant under the integration.

Variations in the roll-angle of free swimming fish is observed to be small, so it is likely that negligible fault is induced by using the roll-angle independent form, $\sigma(\tau)$, of the scattering cross section. In this case

$\sigma'(\alpha,\gamma,\theta,\phi)$

$$= \sigma(2\,\mathrm{Arcsin}(\sqrt{\tfrac{1}{2}(1-\sin\theta \, \cos\alpha \, \cos(\phi-\gamma)-\cos\theta \, \sin\alpha)}) - \tfrac{\pi}{2}) \quad (6)$$

We assume from now on that p is independent of $\phi-\delta$ which is the angle between the orientation of the fish and the transducer. Even if the beam pattern function is not symmetric about the acoustic axis, there should be no reason to believe that this assumption is far from true for transducers designed for fish detection. Therefore we can relative to the transducer in use associate to each fish its echo value $p(\alpha,\beta)$ given by (5) which only depends on the tilt and roll angles in relation to the sea.

Now, we assume that the fishes in the area under consideration is distributed not to near the bottom or transducer level.
Let them have echo functions and echo values $E_i(x, y)$ and $p_i(\alpha_i,\beta_i)$
$i = 1,2,3, \ldots$ respectively.

We call

$$E(x,y) = k \sum_i \ell_i \mathrm{TVG}(R_i) \qquad (7)$$

the areal echo-function and

$$P = \iint E(x,y) \, dxdy \qquad (8)$$

the areal echo-value, where

ℓ_i is the echo-energy picked up by the transducer from fish i after a soundpulse has been transmitted, and R_i is the distance between the transducer and the i-th fish.

In the case of time overlapping echoes, there is no single relation between $E(x,y)$ and $E_i(x,y)$ i = 1, 2, ...
To overcome this, we introduce the following mean values.

Assume there is a joint distribution of tilt and roll angle for each specie and length of fish. For a given fish with distribution $f(\alpha,\beta)$, echo-function $E(x,y,\alpha,\beta)$ and echo value $p(\alpha,\beta)$ we define

$$\bar{E}(x,y) = \int\int E(x,y,\alpha,\beta) \ f(\alpha,\beta) \ d\alpha d\beta$$
and $\hspace{11cm}$ (9)
$$\bar{p} = \int\int p(\alpha,\beta) \ f(\alpha,\beta) \ d\alpha d\beta$$

to be its mean echo function and echovalue respectively.

Tilt and roll angle distributions for free swimming fish have been estimated by means of underwater photographs, and together with measurements of $\sigma(\tau,\rho)$ on a lot of fishes, tables of values proportional to $\bar{p}$ as a function of fish length and specie has been worked out. See Nakken and Olsen [8] . For obtaining exact values of the corresponding function $\bar{p}$, a calibration of the equipment in use is necessary.

Let $\bar{\bar{E}}(x,y)$ and $\bar{P}$ be corresponding mean values for $E(x,y)$ and P respectively, but here the mean is taken over all phase differences of multiple echoes as well as tilt and roll angles.

$E(x,y)$ is strongly time dependent since the fishes almost continuously change their positions and orientations. However, we assume that real fish distributions have the property that the line integral

$$\int E(x,y)ds \rightarrow \int \bar{\bar{E}}(x, y) \ ds$$
and the surface integral $\hspace{9cm}$ (10)

$$\int\int E(x,y)dxdy \rightarrow \int\int \bar{\bar{E}}(x,y)dxdy$$

It is likely that the convergence is fairly rapid when the line or area of integration increases, unless the fishdensity is very small. Further it is evident that $\overline{P} = P$.

Both experiments and theoretical considerations indicate that the relation

$$\overline{E}(x,y) \quad = \quad \sum_i \overline{E}_i(x,y) \qquad\qquad (11)$$

and hence

$$P \quad = \quad \sum_i \overline{P}_i \qquad\qquad (12)$$

hold for fish densities pr unit area up to a certain limit depending on the specie and size distribution of the fishes.

Fish densities where (11) and (12) does not hold have been observed. However, this problem is worked with (see Røttingen [13]) , and it is probable that the so called shadow effect can be compensated for by means of a special transformation of the signal. If so, (11) can be thought of as a a definition of $E(x,y)$. In any case we assume that (11) and (12) is valid in this paper.

The purpose of the echo integrator is to perform line integration of $E(x,y)$ along the course of the ship. Although we assume in this paper that such line integral data can be obtained, we nevertheless look at some of the practical problems.

Let $m(x,y)$ be defined by

$$M(x,y) \quad = \quad m(x,y) \, \Delta s$$

where $M(x,y)$ is the increase in integrator value after a soundpulse transmitted at (x,y) and Δs is the distance sailed between two successive sound pulses. To obtain line integrals of $E(x,y)$, it would be desirable that

$$m(x,y) \quad \propto \quad E(x,y) \qquad\qquad (13)$$

However, energy from reverberation and different noise sources affects the intergrator process system together with $E(x,y)$. See Urick[14] ch. 7 and 8.

Let $I(t)$ be the effect at the input of the treshold unit, at time t after a soundpulse is transmitted at (x,y)

$$I(t) = I_e(t) + I_r(t) + I_n(t) \tag{14}$$

where $I_e(t)$ comes from echoes

 $I_r(t)$ comes from reverberation

and $I_n(t)$ comes from noise sources

Since $\int_{t_1}^{t_2} I_e(t)\,dt \quad \cdot \propto \quad E(x,y)$

where $[t_1, t_2]$ is a time interval containing all arrival times of echoes from fishes, the influence of $I_r(t)$ and $I_n(t)$ is reduced by the treshold unit which cuts away all parts of $I(t)$ with values below a preset treshold setting T.

That is

$$m(x,y) \quad \propto \quad \int_{t_1}^{t_2} u(I(t) - T)\, I(t)\,dt \tag{15}$$

$$\text{where } u(x) = \begin{cases} 1 & x > 0 \\ 0 & x \leq 0 \end{cases}$$

so it is necessary for $E(x,y) \propto m(x,y)$ that

$$\int_{t_1}^{t_2} I_e(t)\,dt = \int_{t_1}^{t_2} u(I(t) - T)\, I(t)\,dt$$

or equivalently $\hspace{8cm}$ (16)

$$\int_{t:I(t)\leq T} I_e(t)\,dt = \int_{t:I(t)>T} (I_r(t) + I_n(t))\,dt$$

Because of the great variability of $I_e(t)$ and $I_r(t)$ (reflections from plankton layers is also classified as reverberation) with position (x, y), (15) cannot be fulfilled generally for any choice of T. In practice, several echo-integrators integrates over each of their disjoint subinterval of $[t_1, t_2]$ and the treshold setting on each integrator is separately adjusted just to cut away most of the noise and reverberation when no echoes are received.

Since the mean effect of single echoes is proportional to the - 4th power of t while the mean effect of reveberation and noise are proportional to the - 2nd power of t and t respectively, the treshold has different properties when integrating echoes from layers of different depths. Some effects of treshold induced bias are studied in Weimer and Ehrenberg[15].

However, we do not take account of the difficulties mentioned above. That is, we assume that the echo integrator system satisfy (13). Then by defining the constant in (1) and (7) by

$$m(x, y) \quad = \quad E(x, y) \tag{17}$$

we can drop the special notation $m(x, y)$.

Specification of the sampling problem and related concepts.

In this paper a fish population is defined to consist of all fishes of a special specie of interest plus fishes of other species mixed with the previous one in such a way that their echo contribution cannot be distinguished from the others and eliminated. For the purpose of acoustic stock estimation, we assume that the fish population is distributed pelagically over an approximately known area and that after all the long distance variations in the fish density can be considered constant during an acoustic survey.

Let the members of the fish population be enumerated and denoted f_1, f_2, ... , f_N.

To each fish there is associated values of several variables like length, biomass, fat content, indicator variable of sex, age groups, maturity etc. of which the sum or mean value over subsets of the population are of interest. We call them individual variables.

Let x be an individual variable with value x_i on f_i, and let C be a subset of the fish population, usually a trawlcatch, a sample and so on. Let us write

$$x(C) \quad = \quad \sum_{i:f_i \in C} x_i \qquad\qquad (18)$$

However, if C_A is the subset of the fish population within a segment A of the sea we write $x(A)$ instead of $x(C_A)$.

Let A_1, A_2, ..., A_n be a stratification of the sea into disjoint strata. The strata will usually consist of sections of the sea containing large scattering layers of fish. General requirements to the strata is that first of all fish density, age distribution and length distribution are less variable within the strata than in the whole population. We associate an areal echo function $E_i(x, y)$ to A_i $\qquad i = 1, 2, ..., n.$

If two or more strata at different depts have overlapping projections on the sea surface, we assume that the echoes from different strata always arrive at the transducer in disjoint time intervals i.e. that their echo functions can be line integrated separately on different integrators.

$$\text{Let} \quad A = \bigcup_{i=1}^{n} A_i$$

We will deal with the problem of estimating $x(A)$, where x is an individual variable, by means of line integral data of $E_i(x, y)$ $i=1, 2, \ldots, n$ and measurements of x and $\bar{p}$ (the mean echo value is also an individual variable) on fishes in several samples taken from trawl catches. The mean echo value $\bar{p}$ is assumed to have a known functional relationship to some measurable quantities on the fish. Until now a relation to fish length have been used.

Some examples of individual variables and corresponding meaning of $x(A)$ are:

x	$x(A)$
Fish biomass on a particular specie, zero on other species	Population biomass for the particular specie.
Indicator of particular specie	Number of fishes of that particular specie
Indicator of one year old fish	Number of one year old fishes

(Note that an indicator is a variable which takes on the values zero and one only).

Without loss of generality we will construct and study estimators $\hat{x}(A_i)$ of $x(A_i)$ $i = 1, 2, \ldots, n$ since these will be assumed independent. Let $a(A_i)$ be the area of the projection of A_i on the sea surface and define

$$\rho_p(A_i) = \frac{\bar{p}(A_i)}{a(A_i)} \quad \text{and} \quad r_x(A_i) = \frac{x(A_i)}{\bar{p}(A_i)} \tag{19}$$

Motivated by the relation

$$x(A_i) = a(A_i) \; \rho_p(A_i) \; r_x(A_i)$$

which follows from (19), we construct the estimator

$$\hat{x}(A_i) = a(A_i) \; \hat{\rho}_p(A_i) \; \hat{r}_x(A_i) \tag{20}$$

where $\hat{r}_x(A_i)$ is an estimator of $r_x(A_i)$ based on catch data and $\hat{\rho}_p(A_i)$ is an estimator of $\rho_p(A_i)$ based on integral data.

It follows from (12) that

$$\rho_p(A_i) = \frac{1}{a(A_i)} \iint_{proj(A_i)} E_i(x,y) dx dy$$

where $proj(A_i)$ is the projection on the sea surface, so $\hat{\rho}_p(A_i)$ is an estimator of the mean of a surface integral based on line integral data, while $\hat{r}_x(A_i)$ is an estimator of a population ratio based on a two stage sample; i.e. trawling and subsampling from each trawl catch.

If $\hat{\rho}_p$ and $\hat{r}_x$ can be considered independent, arguments similar to those used in Raj [12] 1.5 leads to the following relation.

$$MSE(\hat{x}) = a^2 \; MSE(\hat{\rho}_p) MSE(\hat{r}_x) + r_x E(\hat{r}_x) MSE(\hat{\rho}_p) \tag{21}$$

$$+ \; \rho_p E(\hat{\rho}_p) \; MSE(\hat{r}_x) + \; \rho_p r_x \; b(\hat{\rho}_p) \; b(\hat{r}_x)$$

where MSE() and b() stands for mean square error and bias respectively. We have dropped A_i throughout the equation for simplicity.

Further

$$E(\hat{x}) = a \; E(\hat{\rho}_p) \; E(\hat{r}_x) \tag{22}$$

Now the problem arise how to build the estimators $\hat{\rho}_p$ and $\hat{r}_x$ and derive their properties in relation to the survey method and assumptions about the nature of the fish distribution.

We will not treat this problem in detail here, but only put forward some ideas.

First, let us consider $\hat{r}_x$. A set of trawl hauls is taken in each strata. Let the catches within one strata A_i be denoted C_1, C_2, ..., C_t respectively. Although the positions of the hauls is not chosen by any random mecanism, we have reason to believe that

$$\frac{\sum\limits_{1}^{t} x(C_j)}{\sum\limits_{1}^{t} \bar{p}(C_j)} = r_x(\overset{t}{\underset{1}{\cup}} C_j) \tag{23}$$

estimates $r_x(A_i)$ fairly well provided trawl selection bias can be neglected. Let m_j fish be subsampled randomly from C_j $\quad j = 1, 2, ..., t$

Although C_1, C_2, ..., C_t is not a sample of clusters from any cluster partition of the fish population within A_i, the theory of ratio-estimation in two stage sampling may be used with success if the trawl hauls is located sufficient regularity in A_i. Following Raj[12] 6.34, the following estimator is proposed

$$\hat{r}_x = \frac{\sum\limits_{j=1}^{t} \frac{V_j}{v_j} \sum\limits_{k=1}^{m_j} x_{jk}}{\sum\limits_{j=1}^{t} \frac{V_j}{v_j} \sum\limits_{k=1}^{m_j} \bar{p}_{jk}} \tag{24}$$

where v_j and V_j are the volume of the subsample and trawl catch respectively, i.e. $\dfrac{v_j}{V_j}$ is used as subsampling fractions.

x_{jk} and $\bar{p}_{jk}$ are the measurements on fish no. k in the subsample from C_j. Setting the primary unit sampling fraction equal to zero, an estimator for $MSE(\hat{r}_x)$ can be obtained using Raj[12] 6.37.

A more detailed treatment of the problem of estimating $r_x(A_i)$ will be given in a future paper where also the effect of trawl selection is compensated for.

Now, let us pass to the estimator $\hat{\rho}_p(B)$ where B is an arbitrary strata. Intuitively, a number obtained by surfaceintegrating a function on proj(B) defined by $E(x, y)$ on the survey net and interpolation between those values outside the survey net and devide it by a(B), would be a reasonable estimate of $\rho_p(B)$.

One kind of estimator of this form is

$$\int_S \Psi(s) \; E(x(s), y(s))ds \qquad (25)$$

where S is the survey net on proj(B) and s is a distance variable along S. $\Psi(s)$ is a weight function which must be chosen.
It is natural to define $\Psi(s)$ as follows.

Let l(s) be the length of the line (or curve) which consist of exactly those points which has s as the nearest point on S. The function l(s) is easy to evaluate if the survey net consists of a system of straight lines. Now, if

$$\psi(s) = \frac{l(s)}{\int_S l(s)ds}$$

(25) is equal to the mean of a surface integral of a function defined on proj(B) by a kind of interpolation between the survey lines.

However, there is no satisfactory mathematical sampling theory which deals with the problem of estimating a surface integral from observations on a designed system of lines.

In the easy readen article Matérn [6] , Matérn has dealt with this shortcoming of sampling theory which manifests itself in many biological problems.

In order to have a hope of obtaining information about the sampling error when using (25), it is necessary to build a probabilistic model for $E(x, y)$ within each strata which takes acount of known characteristic features of the areal echo functions under consideration. A lot of information about such caracteristic features for several fish populations has been known during acoustic surveys in the past.

In order to formulate a probabilistic model for the areal echo function, we have to define $E(x, y)$ to be the realisation of a random field (measure) on proj(B). For a definition see Doob[1] or Dayley and Vere-Jones [2] . That is, the echo function is a random variable for each point (x, y) and has moments denoted by

$$\mu(x, y) \quad = \quad E(E(x, y))$$

$$c(x_1, y_1; x_2, y_2) \quad = \quad Cov(E(x_1, y_1) , E(x_2, y_2))$$

(26)

Even $\qquad \rho_p(B) \quad = \quad \dfrac{1}{a(B)} \quad \iint\limits_{proj(B)} E \ (x, y) dxdy$

is a random variable under the above assumptions, but we assume that the strata is large enough to make its varians negligible i. e.

$$\rho_p(B) \quad = \quad E(\rho_p(B)) \quad = \quad \frac{1}{a(B)} \iint\limits_{proj(B)} \mu(x,y) \ dxdy$$

Let $\hat{\rho}_p(B)$ be given by (25).

Then

$$E(\hat{\rho}_p(B)) \quad = \quad \int\limits_{S} \Psi(s) \ \mu \ (x(s), y(s)) ds \tag{27}$$

and

$$E(\hat{\rho}_p(B) - \rho_p(B))^2 = \frac{1}{a^2(B)} U - \frac{2}{a(B)} V + W + \Delta \tag{28}$$

where

$$U = \iiiint\limits_{\text{proj(B)}^2} c\,(x_1, y_1; x_2, y_2)dx_1 dy_1 dx_2 dy_2$$

$$V = \iiint\limits_{\text{proj(B)} \times S} \Psi\,(s)\ c\,(x_1, y_1; x_2(s), y_2(s))dx_1 dx_2 ds$$

$$W = \iint\limits_{S^2} \Psi\,(s_1)\ \ \Psi\,(s_2)\ c\,(x_1(s_1), y_1(s_1); x_2(s_2), y_2(s_2))ds_1 ds_2$$

and $\quad \Delta\ =\ (E(\,\hat{\rho}_p(B)) - E(\,\rho_p(B)))^2$

If Δ can be neglected, which is reasonable for a well chosen Ψ , we see that the mean square error (28) is expressed by the covariance function alone.

Unfortunately there does not exist as far as the author knows any estimator for the measure of error (28). Matérn has worked with the problem in Matérn [4] and [5] for the case of an isotropic random field and has studied the properties of some quadratic forms as estimators when the observations are taken along lines or at points in the plane.

In any case it is important to try to estimate the correlation function $c(x_1, y_1; x_2, y_2)$. On the survey lines $E\,(x(s), y(s))$ is a time series, and so, the covariance function may be estimated there under certain mathematical assumptions. See Jenkins and Watts [3] 5.3. If a rough estimate of the covariance function is obtained by estimation and smoothing on the survey lines and interpolating between them, it could be used in (28) to calculate an estimate of the sampling error of (25).

However, if this method or another proves to be useful for estimating MSE $(\hat{\rho}_p)$, the question which still remains unsolved is this:

How shall we design a survey net and distribute the trawl hauls to obtain the most presize estimates for a given amount of cost or survey time? There exists very little theory on design problems mainly because it is a difficult and new branch in applied mathematics. See Ylvisaker [16] where the design of points of observation in the plane is studied under the assumption of a known correlation function. However, the mathematical problems arising in modelling acoustic estimation methods of fish populations are general and in no way restricted to this field. For ideas on modelling spatial patterns of biological populations and for further references see Pielou [10] and Patil, Pielou, Waters [9] . In any case, the usefulnes of the ideas in this paper rests on the existence of the relation (11), or in other words, the equipments capability of collecting values of the right side of (11). Further the scattering cross section of free swimming fish may depend on several factors in addition to tilt and roll angle, which is not possible to observe neither from the echoes nor on the fishes in the trawl catches. See articles in [11] . This cause no further difficulties as long as the mean echo value over all such factors can be related to observable quantities on the catched fishes. If this mean echo value of single fish, however, happens to be related to observable factors like depth, time etc. in a way which cannot be compensated for by the signal processing system alone, the stratification must be carried out to give approximately equal effect of these factors throughout each strata.

Finally, to carry out a useful precision increasing stratification of the sea is an art which calls for good knowledge of fish population behavior (how fish of different sizes mixes, internal migration etc.) as well as understanding of the theoretical results in formation of strata (see Raj [12] 4.6).

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