A Method of estimating Mortality in Fish Stocks

by

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I. Theory

Presuming a constant yearly recruitment (N) and a constant yearly survival rate (x), the magnitude of the stock may be written:

$$Sn = N + Nx + Nx^2 = \ldots + Nx^{n-1} = N\frac{1 - x^n}{1 - x} \qquad (1)$$

where N, Nx, and so forth is the strength of the 0 - group, the one year group, and so on up to the year group (n - 1).

Equation (1) may also be written:

$$Sn = N\frac{1 - x^p}{1 - x} + N\frac{x^p - x^q}{1 - x} + N\frac{x^q - x^r}{1 - x} + N\frac{x^r - x^n}{1 - x} \qquad (2)$$

$$(100 = \quad a \quad + \quad b \quad + \quad c \quad + \quad d \quad) \qquad "(2)"$$

If now the stock in question is adequately sampled, the percentage strength of the age groups 0 to (p - 1), p to (q - 1), q to (r - 1), and r to (n - 1) will be known. Let these be denoted by a, b, c, and d respectively "(2)". The number of terms in the right hand side of (2) may be chosen arbitrarily. When four terms are preferred here, it is only because this is a convenient form when comparing with other methods.

From (2) may be constructed another equation:

$$\Pi \frac{x^p - x^q}{1 - x} \ / \ \Pi \frac{x^p - x^n}{1 - x} = b \ / \ b + c + d \tag{3}$$

Since $0 < x < 1$, x^n will normally be a smaller number compared to x^p, and may be ignored. Equation (3) takes then the form:

$$x^{q-p} = \frac{c + d}{b + c + d} \quad \text{or} \quad x = \sqrt[q-p]{\frac{c + d}{b + c + d}} \tag{4}$$

This simplified expression cannot be used when n is small, in which case the whole method breaks down anyway. But for long-lived species the formula (4) should be applicable, insofar the basic assumption are approached to some extent.

It will also be realized that if x is large, say about 0.8, this bias introduced by omitting x^n will not be negligible. In this case the equation:

$$bx^{n-p} - (b + c + d) x^{q-p} + (c + d) = 0 \tag{5}$$

should be used. This expression is also a rearrengemant of (3). The difference will, however, be small since a high survival rate (x) also means a high age (n).

The same general idea can be applied to develop formulas expressing the relationship between the life span(n) and the mortality (x), in which case also small values of n will be included. But this question will not be considered here.

II. Application

It is a well known fact that the **strength** of the year classes may vary considerabely in fish stocks, and the yearly recruitment is seldom, if ever, a constant. If several years of observation is at

hand, the values may be averaged and the disturbing effects of varying recruitment will be damped.

Einar Lea has calculated the survival rate of the Norwegian Winter Herring based on age readings from 1907 to 1928. (Lea, 1929). Lea's algebraic method gives: $x = 0.81$ comprising the age classes 7 - 17 years. Lea does not present his basic material, but the age composition can be found in a paper by Oscar Sund (1943). In Table 1 is presented the average age frequency distribution for the period 1907 to 1928 based on the data given by Sund.

Plotting the natural logarithms to the values in Table 1, (7 - 17 years), and fitting a straight line, it will be found that the slope is -0.266 [1]. This means that the yearly survival coefficient is $x = e^{-0.266} = 0.769$ calculated by this second method. (Logarithmic method, Fig. 1).

If in equation (2) $p = 7$, $q = 10$, $r = 18$, and $n = 24$, the value of a is 375 (the sum of the percentages up to 6 years). Similarly, b = 345 (7 - 9 years), c = 270 (10 - 17 years), and d = 10 (18 - 23 years). (Table 1). The figures p, q and r may be chosen arbitrarily. Since, however, this new method is put to test on the same data Lea used for his calculations, the choice of $p = 7$ and $r = 18$ is obvious. The choice of q should be made so that the root extraction will be simple. With (q - p) like 3 or 4, the operation may be readily made on an ordinary slide ruler. With the figures given, (4) yields:

$$x = \sqrt[3]{\frac{280}{625}} = 0.766 \tag{4}$$

and (5) takes the form:

$$345x^{17} - 625x^3 + 280 = 0 \tag{5}$$

[1] Calculations by slide ruler, fitting by eye.

This equation has a solution for $0 < x < 1$ of $x = 0.77$ [1]: But Lea has excluded all fish over 17 years in his calculations. If the same is done here, the equation will be:

$$345x^{11} - 615x^3 + 270 = 0 \qquad (6)$$

and in this case: $x = 0.78$ (Fig. 2). If the simplified formula (4) is used, the result will be $x = 0.76$, which shows that the term $345x^{11}$ is not negligible for such high values of x when the oldest age groups are excluded.

Another commercial species for which age data are available for a series of years, is the arctic cod which spawns in the Lofoten area. For this fish Rollefsen (1933) has introduced readings of spawning zones in the otoliths and instead of age distribution, one gets in this case a distribution of spawning groups. These data may, of course, be treated in the same way as the age distribution as far as calculations of mortality rates are concerned. Here is used the data published in Rap. Proc-Verb. Vol. 136, 1954. (Aasen, 1953). In Table 2 is presented the average distribution in per mille of the spawning groups of female cod in Lofoten for the years 1932 - 52.

The Lea method, applied on the spawning groups I - VIII, gives $x = 0.506$.

The "power series method" is in this case very simple. The three first spawning groups constitute 882 per mille. Going back to the original formula this may be written:

$$1 - x^3 / 1 - x^{13} = 882/1000$$

in which case x^{13} may be ignored for such a high value of n and low value of x. This gives:

$$1 - x^3 = 0.882 \quad \text{or} \quad x = \sqrt[3]{0.118} = 0.491$$

[1] Solved graphically by a method illustrated in Fig. 2.

Using the "logarithmic method" the result will be [1] $x = 0.495$. (Fig. 3).

Both scales and otoliths may be difficult to read for older ages. With the "power series method", however, where certain intervals of age groups are pooled, it will be sufficient if the individual reading can be carried up to a certain age $(q - 1)$ with reasonable exactitude. What is above that age goes into the last group. For the first group is chosen the age $(p - 1)$ where full recruitment is attained. It will be realised that also in this case equation (4) is valid, and this is perhaps the greatest merit of the new method.

The Atlantic Mackerel from the North Sea area presents a case as described above: [x] The otoliths are generally hard to read clearly above eight years $(q = 9)$ and [p] full recruitment is attained at the age of five years $(p = 5)$.

Reliable Norwegian data for the age distribution are available only for the last three years, 1967 - 69. (Table 3). In some cases the otoliths from the larger fish are impossible to read even to 8 years of age. Rather than discarding these, they have been grouped according to the rule that for $L > 38$ cm the age is over 8 years, and for $L \leq 38$ cm the age is between 5 and 8 years.

For 1967 the condensed data from Table 3 show:

0 - 4	5 - 8	> 8
157	512	331

$\qquad$; $\qquad x^4 = \dfrac{331}{843} = 0.393$; $\quad x = 0.791$

similarly 1968:

0 - 4	5 - 8	> 8
533	312	155

$\qquad$; $\qquad x^4 = \dfrac{155}{467} = 0.332$; $\quad x = 0.761$

1) Calculations by slide ruler, fitting by eye.

and 1969:

$$\begin{array}{c|c|c} 0 - 4 & 5 - 8 & > 8 \\ \hline 334 & 410 & 256 \end{array} \quad ; \quad x^4 = \frac{256}{666} = 0.384 \quad ; \quad x = 0.789$$

For the period 1967 - 69, the average survival rate is:

$$\begin{array}{c|c|c} 0 - 4 & 5 - 8 & > 8 \\ \hline 341 & 412 & 247 \end{array} \quad ; \quad x^4 = \frac{247}{659} = 0.375 \quad ; \quad x = 0.784$$

III. Conclusions

The three methods for calculation of the survival rate show reasonable correspondence when applied to the age data of the Norwegian Winter Herring in the period 1907 - 28. It will be seen that Lea's method gives the higher value of x [0.81]. The new method, which may be termed "power series method", gives practically the same result as the "logarithmic method" 0.769 both for the abbreviated formula [0.766] and for equation (5) [0.77]. For equation (6) the value of x is a little higher [0.78].

For the case of the Lofoten cod, it will be seen that the results are in fair agreement when the three methods are compared. Here too the Lea method gives the highest value of x [0.506]. The "power series method" [0.491] and the "logarithmic method" [0.495] give practically the same results, as was also the case for the herring material.

The two examples for the herring and cod data show that the "power series method" is applicable for calculating survival rates for long lived species where long series of ageing are at hand. For the North Sea mackerel there are only three years of data available in the Norwegian material. But it is evident that these three years

show reasonable correspondence in the survival rates calculated by the new method. For this kind of data, both the "logarithmic method" and Lea's method break down, at least in their original forms.

The high survival rate found for the mackerel, is rather surprising. Accepting, however, the evidence, it will mean that the mackerel reach a high age, say up to 24 -25 years. It is fairly obvious, that when the ring net fishery for mackerel started in the North Sea area, there was an accumulated stock. Whith such a high survival rate, this stock should be expected to be large. That this is so, is borne out by the fact that according to the official Norwegian fishery statistics, about 2.5 million tons have been landed in the period 1965 - 68. Representative samples drawn from the commercial catches when the intensive fishery started , should more or less show the natural survival rate.

But it will be realized, that the same would be the case if there was no recruitment to the stock, irrespective of the stock level. An inspection of Table 3 shows that the last strong year class to enter the adult stock was the 1962 year class.

I may be concluded then, that the calculated average survival rate. 0.784, is approaching the natural survival rate for the North Sea mackerel because the effects of the heavy fishing is counteracted in the formula by failing recruitment . This knowledge may be useful at a later stage, when the recruitment is "normalized", or if, f.i. tagging data will provide reliable estimates of the exploitation rate.

References

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Table 1

Average age frequency distribution (°/oo) of the Norwegian Winter Herring in the years 1907 – 1928.

Year	A g e																					
	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
1907 – 28 Σ	4,1	71,4	150,2	148,8	128,6	114,3	102,4	76,0	49,2	39,8	33,2	26,7	20,3	15,1	9,9	5,9	2,8	0,6	0,6	0,3	0,7	1000,9

Table 2 Average distribution (°/oo) for the spawning groups of Lofoten cod ($\female$) in the years 1932 – 1952.

Year	Spawning groups													Total
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	Total
1932 – 52	473,6	277,0	131,4	52,4	28,3	15,9	9,9	5,5	2,5	2,2	1,0	0,4	0,2	1000,3

Table 3 Age groups (°/oo) North Sea Mackerel. (No age reading: $U_1 \leqq 38$ cm; $U_2 > 38$ cm).

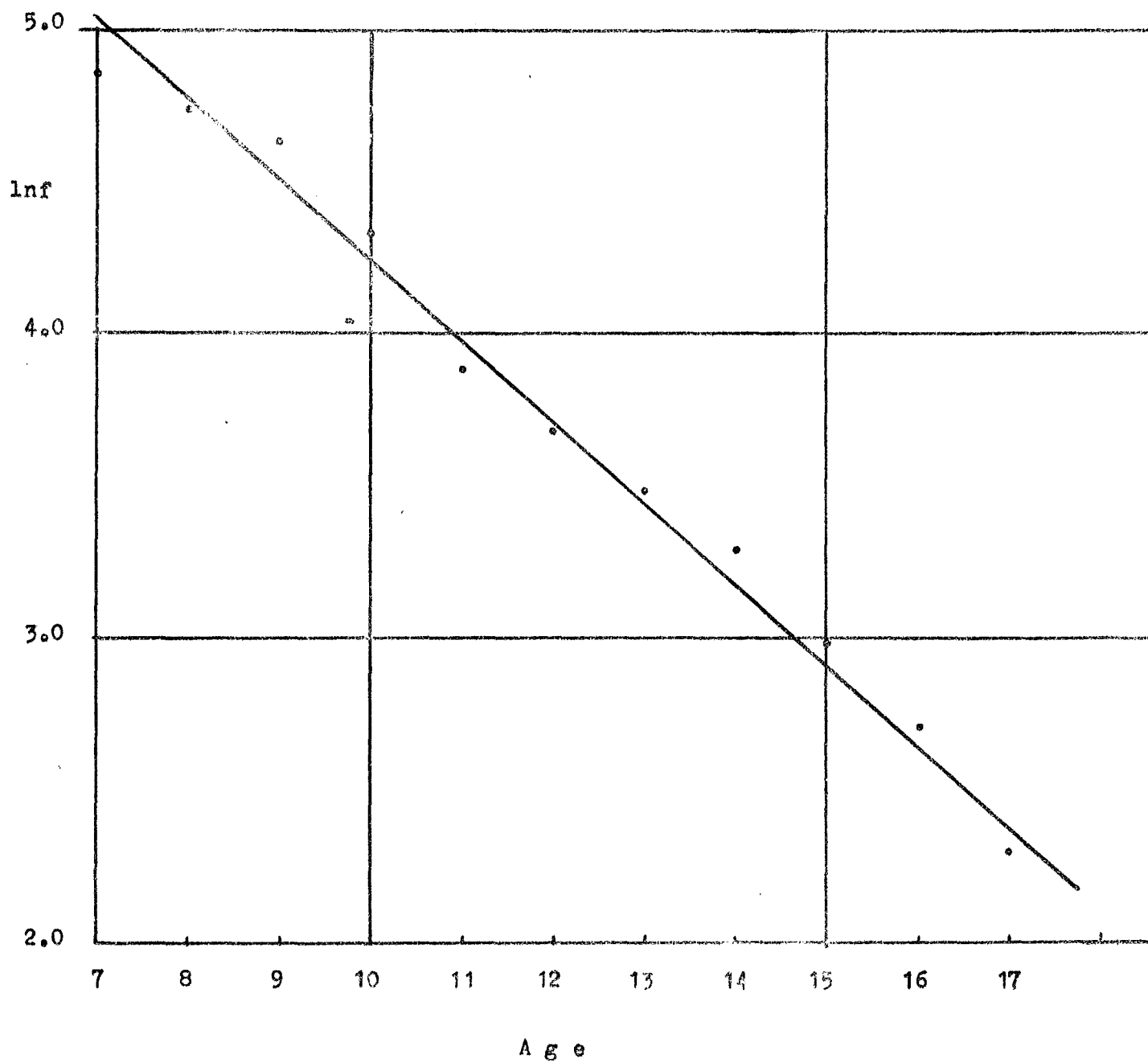| Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	U_1	U_2	Total
1967	–	7,7	78,5	50,0	20,8	243,1	43,8	31,5	39,2	35,4	16,9	20,0	24,6	2,3	0,8	–	–	–	154,6	230,8	1000,0
1968	–	61,2	302,0	126,5	42,9	14,3	120,4	16,3	16,3	18,4	38,8	10,2	18,4	6,1	2,0	2,0	–	2,0	144,9	57,1	999,8
1969	–	–	7,7	151,3	174,4	59,0	15,4	169,2	46,2	10,2	23,1	12,8	7,7	15,4	15,4	–	–	–	120,5	171,8	1000,1
Mean	–	23,0	129,4	109,3	79,4	105,5	59,9	72,3	33,9	21,3	26,3	14,3	16,9	7,9	5,1	0,7	–	0,7	140,0	153,2	1000,1

Fig. 1   Logarithmic method.   ● Natural
          logarithms to the frequencies in
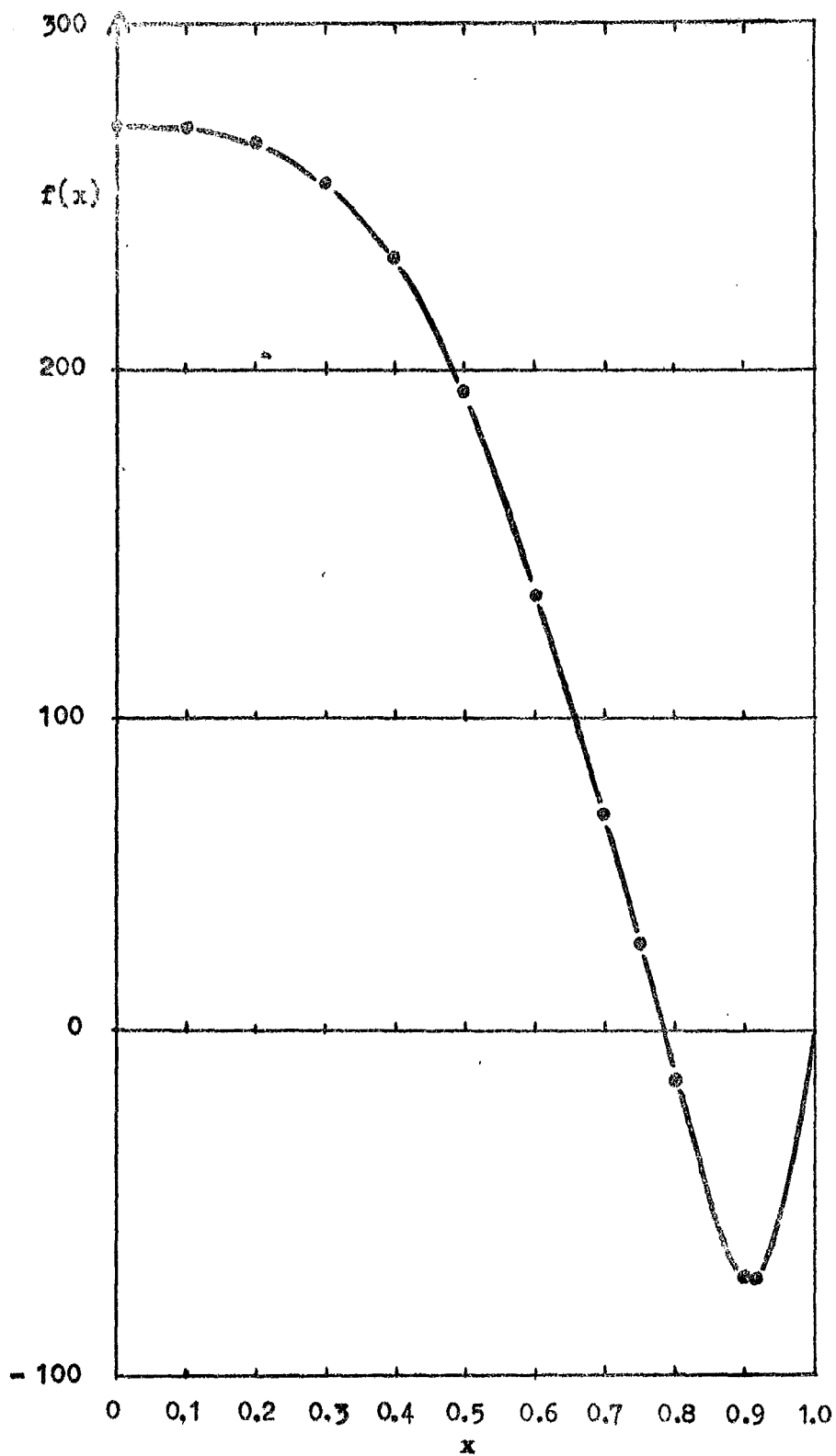   Table 1.   z = 0.266 , x = 0.769

Fig. 2   Power series method.

● Calculated values of: $f(x) = 345 \, x^n - 615 \, x^3 + 270$

   $f(x) = 0$ for $x = 0.78$

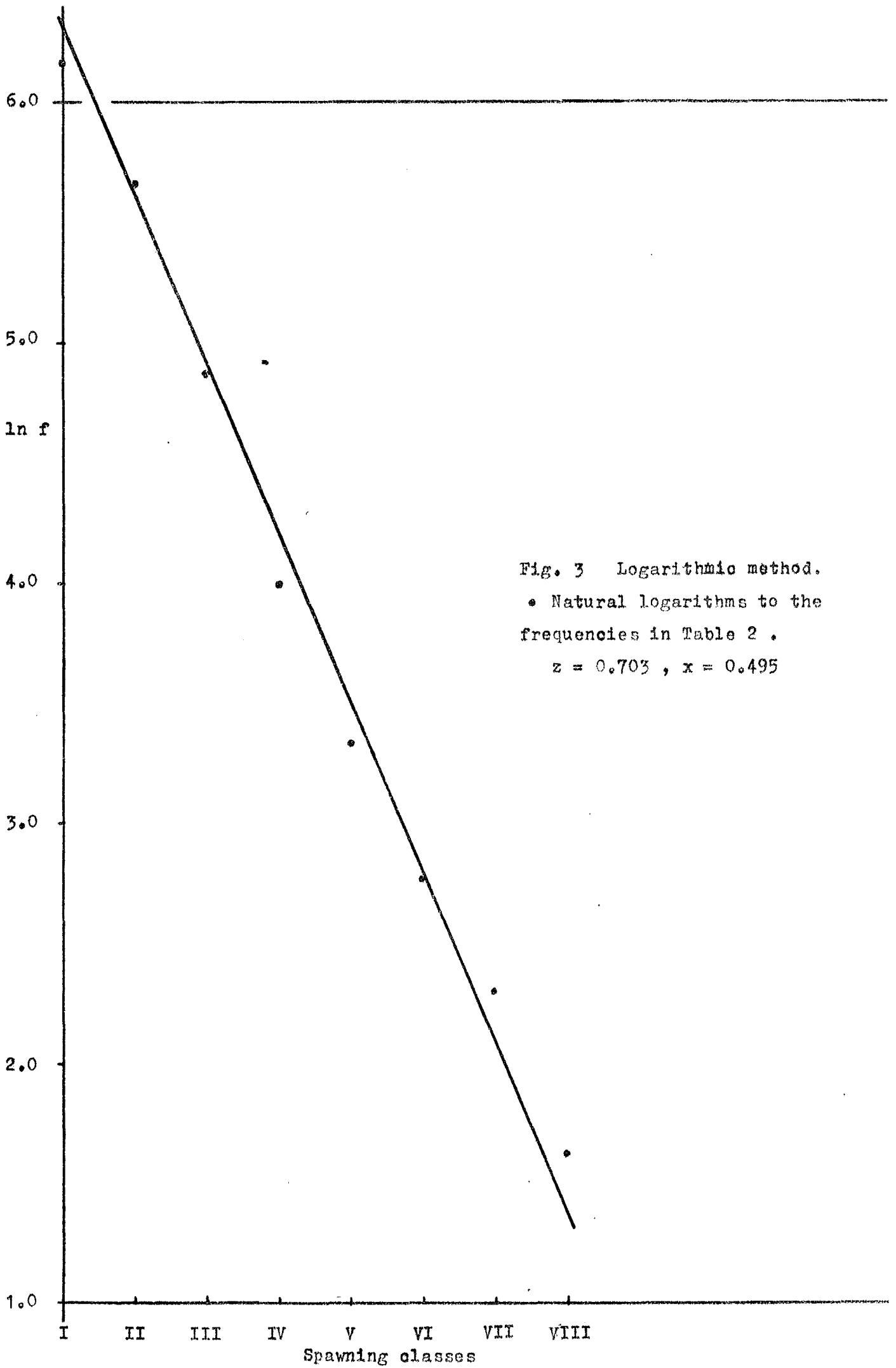 $f'(x) = 0$ for $x = 0.915$

   $f''(x) = 0$ for $x = 0.75$

Fig. 3   Logarithmic method.
• Natural logarithms to the
frequencies in Table 2 .
$z = 0.703$, $x = 0.495$