International Council for the Exploration of the Sea

C.M. 1987/B:2
Sess. O
Fish Capture Committee

DEPENDENCE OF EQUIVALENT BEAM ANGLE ON SOUND SPEED

by

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ABSTRACT

The equivalent beam angle is computed numerically, according to its rigorous definition, for each of two transducers. These are those used with the SIMRAD split-beam echo sounders at 38 and 70 kHz. The results are expressed as functions of the medium sound speed.

RÉSUMÉ: INFLUENCE DE LA VITESSE DU SON SUR L'ANGLE ÉQUIVALENT DU FAISCEAU

L'angle équivalent du faisceau est calculé numériquement, selon sa définition rigoureuse, pour chacun des deux transducteurs, utilisés avec les sondeurs SIMRAD à faisceau scindé en 38 et 70 kHz. Les résultats sont exprimés sous la forme de fonction de la vitesse moyenne du son.

INTRODUCTION

The equivalent beam angle is a key quantity in both the echo-counting and echo integration methods of measuring fish density (Simmonds 1984a). This is evident from the basic mathematical expressions for fish density. These normalize a count or echo integral to the observation or sampling volume. Since this volume is directly proportional to the equivalent beam angle, the dependence of fish density on it is first-order (Foote et al. 1987). An error in the equivalent beam angle will also have a first-order effect on estimates of fish density. This is why the angle is studied here.

The particular aim of this work is to specify the dependence of the equivalent beam angle on the medium sound speed. This will indicate the need for adjusting the angle value to changing hydrographic conditions.

METHOD

The equivalent beam angle is defined as the integral of the product of farfield transmit and receive beam patterns over all space. If the respective beam patterns are written $b_T(\theta,\phi)$ and $b_R(\theta,\phi)$, where (θ,ϕ) is the general spatial direction, then the equivalent beam angle Ψ is (Urick 1975)

$$\Psi = \int_0^{2\pi} \int_0^{\pi} b_R(\theta,\phi)\; b_T(\theta,\phi)\; \sin\theta \; d\theta \; d\phi \quad . \tag{1}$$

This definition is quite general, for it allows for the possibility of using separate transmitting and receiving transducers oriented in different directions.

For ordinary applications in fisheries acoustics, the same transducer performs both functions. In the absence of inter-element coupling, which is usually a low-frequency effect of little consequence for typical survey frequencies and transducer sizes, $b_T=b_R$. This is the case envisaged here.

The beam patterns of the subject SIMRAD split-beam transducers have been defined in an earlier report to the Council Meeting (Foote 1986). Both the 38 and 70 kHz transducers are examples of planar arrays of identical circular elements. For a like array of n elements of constant radius a, the beam pattern is defined for $\theta \in [0,\pi/2]$ as

$$b(\theta,\phi) = \left| \frac{2\, J_1(ka \sin\theta)}{ka \sin\theta} \frac{1}{n} \sum_{j=1}^{n} \exp(i\underline{k}\cdot\underline{r}_j) \right|^2 \quad , \tag{2}$$

where $\underline{k}$ is the wavevector and $\underline{r}_j$ is the position of the center of the j-th element. The wavevector specifies the field direction $k=\underline{k}/k$ or point $(\sin\theta \cos\phi,\ \sin\theta \sin\phi,\ \cos\theta)$ on the unit sphere. Its magnitude is $k=2\pi/\lambda=2\pi\nu/c$, where λ is the wavelength, ν is the frequency, and c is the medium sound speed.

Details of the geometry, or quantities a and $\{\underline{r}_j, j=1,2,\ldots,n\}$, are given as follows.

<u>38 kHz transducer</u>. a=17.5 mm. n=68. The elements are packed as densely as possible in a hexagonal array. The number of elements per row follows the sequence: 6, 7, 8, 9, 8, 9, 8, 7, 6. The middle row thus lacks the end elements of the ordinary hexagonal single-beam transducer.

<u>70 kHz transducer</u>. a=8.5 mm. n=36. The array geometry is square. The inter-element distance along rows and columns as measured between element centers is 18 mm.

In order to study the dependence of Ψ on c, the integration in equation (1) is performed by means of a general multidimensional integration routine, D01FCF, in the NAG Library (1984). Constraints on the computations were that at least 1000 points be used in each integration and that the relative accuracy be better than 0.001. The beam pattern specified in equation (2) was computed for the particular parameters of the 38 and 70 kHz transducers in a function subroutine called by D01FCF. The beam pattern was assumed to vanish for $\theta > \pi/2$.

RESULTS

These are given in the table.

Equivalent beam angle Ψ as a function of medium sound speed c for two SIMRAD split-beam transducers.

	38 kHz transducer		70 kHz transducer	
c (m/s)	Ψ	10 log Ψ	Ψ	10 log Ψ
1400	0.00859	-20.66	0.01550	-18.10
1410	0.00871	-20.60	0.01570	-18.04
1420	0.00883	-20.54	0.01590	-17.99
1430	0.00896	-20.48	0.01612	-17.93
1440	0.00908	-20.42	0.01634	-17.87
1450	0.00921	-20.36	0.01657	-17.81
1460	0.00934	-20.30	0.01679	-17.75
1470	0.00947	-20.24	0.01702	-17.69
1480	0.00960	-20.18	0.01726	-17.63
1490	0.00973	-20.12	0.01749	-17.57
1500	0.00986	-20.06	0.01772	-17.52
1510	0.00999	-20.00	0.01797	-17.45
1520	0.01012	-19.95	0.01821	-17.40
1530	0.01026	-19.89	0.01845	-17.34
1540	0.01039	-19.83	0.01869	-17.28
1550	0.01053	-19.78	0.01894	-17.23
1560	0.01066	-19.72	0.01918	-17.17

DISCUSSION

The results show a nearly constant 0.06 dB change in the quantity 10 log Ψ for each change in c by 10 m/s. This can be understood through small-angle approximations to Ψ. For circular or rectangular arrays, with θ_a and θ_b measuring the angles in degrees from the axis to the -3 dB levels of one-way beam patterns in perpendicular planes or cuts,

$$10 \log \Psi \doteq 10 \log (\theta_a \theta_b) - 31.6 \quad , \tag{3a}$$

as given by Urick (1975) and used by, among others, Ona and Vestnes (1985). Another approximation that is in the same spirit but is applicable to more general two-dimensional arrays is

$$10 \log \Psi \doteq 20 \log \theta_{ave} - 31.6 \quad , \tag{3b}$$

where θ_{ave} is the average angle at -3 dB level, as measured from the axis, over all directions.

Evaluation of Ψ through equations (3a) and (3b) yields similar results. The values from equation (3a) are generally 0.02 dB less than the respective values from equation (3b) for the 38 kHz transducer and 0.08 dB less for the 70 kHz transducer. Both approximations give values for Ψ which are about 0.1 dB higher than those determined by numerical evaluation of equation (1).

At the -3 dB angles for circular and rectangular arrays,

$$kb \sin \theta = \text{constant} \quad , \tag{4}$$

where b is a characteristic dimension of the array. For the generally large values of kb for typical transducers in use on survey vessels,

$$\theta \propto (kb)^{-1} \quad , \tag{5}$$

hence

$$\theta \propto c \quad . \tag{6}$$

Thus the change in Ψ due to a change Δc in c is

$$10 \log \Psi_2 - 10 \log \Psi_1 \doteq 20 \log \frac{c_2}{c_1} \doteq 20 \log \left(1 + \frac{\Delta c}{c_1}\right) \quad . \tag{7}$$

For small Δc,

$$20 \log \left(1 + \frac{\Delta c}{c}\right) \doteq \frac{20}{\ln 10} \frac{\Delta c}{c} \doteq 8.686 \frac{\Delta c}{c} \quad . \tag{8}$$

Hence, if Δc=10 m/s for the sound speed c=1475 m/s,

$$\Delta(10 \log \Psi) \doteq \frac{8.686}{147.5} \doteq 0.059 \quad . \tag{9}$$

The observation from the tabulated results is thus confirmed.

It is also to be noted that the listed values are absolute. These are moreover consistent with the manufacturer's test measurements. Helge Bodholt, SIMRAD Subsea, has reported on test measurements of Ψ for the 38 kHz split-beam transducer in a personal communication. For the last 17 transducers measured up to the time of the inquiry, the average value was -20.5 dB. The datum for each transducer was based on measurements of the beam pattern in each of two orthogonal planes and use of the small-angle formula in equation (3a). The accuracy of the laboratory measurements of Ψ is estimated by Bodholt to be ±0.5 dB.

Additional confirmation of the absolute values is provided by Hood (1987) and Reynisson (1987). Hood measured a split-beam transducer in its towed-body housing in a laboratory tank. Use of the -3 dB beam pattern angles in equation (3a) gives $10 \log \Psi=-19.7$ dB. Reynisson's measurements of a hull-mounted split-beam transducer when driven solely by the internal transmitter determines $10 \log \Psi=-20.1$ dB.

The value of Ψ used by the Institute of Marine Research, Bergen, with its split-beam transducers is -19.6 dB. Differences between this and laboratory measurements and theoretical computations might be attributed to the effects of mounting and housing (Simmonds 1984b, Ona and Vestnes 1985). Can these be computed?

It is important to note that repetition of the present numerical computations over much more restricted ranges of θ than the present range $[0,\pi/2]$ gives almost indistinguishable results. In particular, definition of non-vanishing b by equation (2) for $\theta \in [0,\pi/18]$ gives the same results to within 0.01 dB for the 38 kHz transducer. In the case of the 70 kHz transducer, the results agree to within 0.03 dB for $\theta \in [0,\pi/9]$. Thus whatever the effects of the mounting and housing are, these must be significant within the central part of the beam, i.e., in the main lobe.

CONCLUSIONS

The present theoretical computations specify the dependence of the equivalent beam angle Ψ for two SIMRAD split-beam transducers on the medium sound speed c in meters per second through these equations: $10 \log \Psi= -20.66 + 0.0059(c-1400)$ for the 38 kHz transducer and $10 \log \Psi=-18.10 + 0.0059(c-1400)$ for the 70 kHz transducer. The coefficient of the sound speed term, 0.0059, is exactly that expected from simple theoretical considerations.

Since Ψ increases by 0.06 dB for increases in c by 10 m/s, ordinary cruise applications will not require adjustment of Ψ from the initial value. This should, however, reflect the large-scale hydrography of the survey region, e.g., sea water or fresh water, Arctic or tropical conditions, given a reference value of Ψ for the transducer when configured for use. Present work indicates that Ψ can be determined to within ±0.1 dB (Simmonds 1984a, Ona and Vestnes 1985, Reynisson 1985, Hood 1987), and this should be the aim of researchers.

Other influences on Ψ, such as those of ambient noise or reverberation in masking weaker targets, require attention.

ACKNOWLEDGEMENT

E. Ona is thanked for reviewing the manuscript.

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