

SIMPLE CALIBRATION TECHNIQUE FOR THE SPLIT-BEAM ECHO-SOUNDER

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ABSTRACT

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The split-beam echo-sounder is used to measure the target strength of fish. Calibration of this instrument requires sensitivity measurements to be made throughout the cross-section of the acoustic beam. It is necessary to measure the sensitivity variation as well as the mean value. The indicated target strength distribution is broadened if the sensitivity changes significantly with the target direction. The theory of this effect is developed. A calibration technique is described which requires few measurements to achieve good accuracy. For the ES400 echo-sounder, measurements at 30 points in the beam determine the mean sensitivity to ± 0.5 dB or better. The sensitivity measurements of a particular transducer and echo-sounder varied over a 3 dB range. Reasons for this unexpected variation are discussed.

INTRODUCTION

The technique of echo-integration is widely applied in pelagic fish surveys (MACLENNAN and FORBES, 1987). The echo-integrator measurement is proportional to the quantity of detected fish. However, the constant of proportionality depends upon the target strength (TS) or the equivalent acoustic cross-section (σ) of the individual fish. In the case of fish targets, the TS varies with the species, the body length and many other factors. Within the

fish population to be surveyed, we expect to find a distribution of target strengths which must be determined experimentally.

Ideally, the TS should be determined for wild fish in the sea, from measurements made during the survey, but it is only recently that sonar equipment capable of such measurements has become available. One such instrument is the split-beam echo-sounder (FOOTE *et al.*, 1986). It compensates the received echo for transmission losses and the transducer beam-function so that, in the case of an isolated target, the echo energy is proportional to the target strength.

There are well-established methods for calibrating the conventional echo-sounder. For the on-axis sensitivity, FOOTE *et al.* (1987) recommend the standard target technique, in which a metal sphere of known TS is suspended on three twines below the transducer. The twine lengths are adjusted to obtain the maximum echo-amplitude, which corresponds to the on-axis position. This measurement is straightforward; it is now performed routinely on sea-going vessels.

It is also necessary to know the beam-function of the transducers used in fish surveys. Present techniques for measuring the beam-function require special apparatus and are not suitable for routine calibration work (SIMMONDS, 1984). In the case of the split-beam echo-sounder, the on-axis sensitivity is of little significance by itself. It is necessary to determine the sensitivity over the active part of the beam cross-section.

In this paper, we describe a simple technique for calibrating the split-beam echo-sounder. The apparatus is the same as that used to calibrate conventional echo-sounders by the standard target method. An example is described from measurements performed on the SIMRAD ES400 split-beam echo-sounder which is installed on the Norwegian research vessel «G.O. Sars». Few measurements are required to obtain good accuracy, and our technique is suited to calibration work both in the field and at sea. The main requirement is that the ship should be anchored in sheltered water at least 25 m deep.

THE SPLIT-BEAM PRINCIPLE

The transducer is divided into four electrically independent sections (Fig. 1). The signal at the transducer terminals depends upon the scattering strength of the target, its range and direction. Conventional time-varied gain in the receiver removes the range-dependence of the signal. The target direction is determined by the signal phase differences between paired sections. This information is used to compensate for the transducer beam-pattern. Thus the effective sensitivity of the instrument (also known as the beam function) is nominally constant for all target directions.

The instrument accepts signals only from single (isolated) targets, using

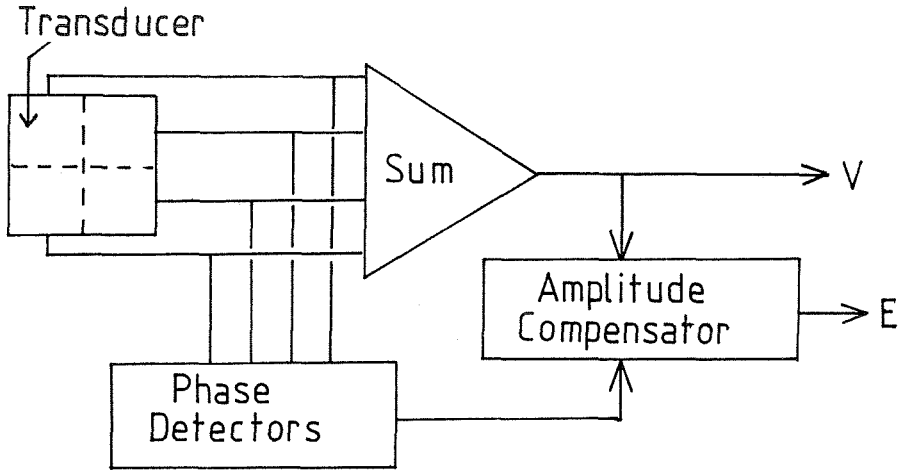


Fig. 1. Principle of the split-beam echo-sounder. The combined signal V varies with the target direction as for a conventional single-beam transducer. The phases of signals from the four transducer sections are used to compensate for the transducer beam-function. Thus the output signal E is nominally independent of target direction.

an echo-duration criterion to identify those signals which are formed from the superimposed echoes of multiple targets. Signals from targets distant from the acoustic axis, where the transducer sensitivity is low, are also rejected. In the case of the ES400 echo-sounder, the «acceptance cone» includes all target directions up to five degrees from the transducer axis.

The calibration is required firstly to measure the transducer sensitivity and receiver gain, and secondly to monitor changes in the sensitivity with target direction. In theory, knowledge of the beam-function should allow exact compensation for directional changes in sensitivity. In practice, however, the beam-function of the transducer may differ from the theoretical prediction, and experimental measurement of the compensated beam-function is a necessary part of the calibration (SIMMONDS, 1984).

THEORY

Consider first the case of a single target whose acoustic cross-section is σ . The target is in direction \hat{f} with respect to the transducer at the origin (Fig. 2), while θ is the angle between \hat{f} and the acoustic axis \hat{z} . It is assumed that exact time-varied gain is applied in the receiver to remove the range-dependence of the echo energy (MACLENNAN, 1987). Thus the output signal E is independent of the target range, but it is weakly dependent on the target direction, in proportion to the compensated beam function $b(\hat{f})$.

$$E = b(\hat{f}) \sigma \quad (1)$$

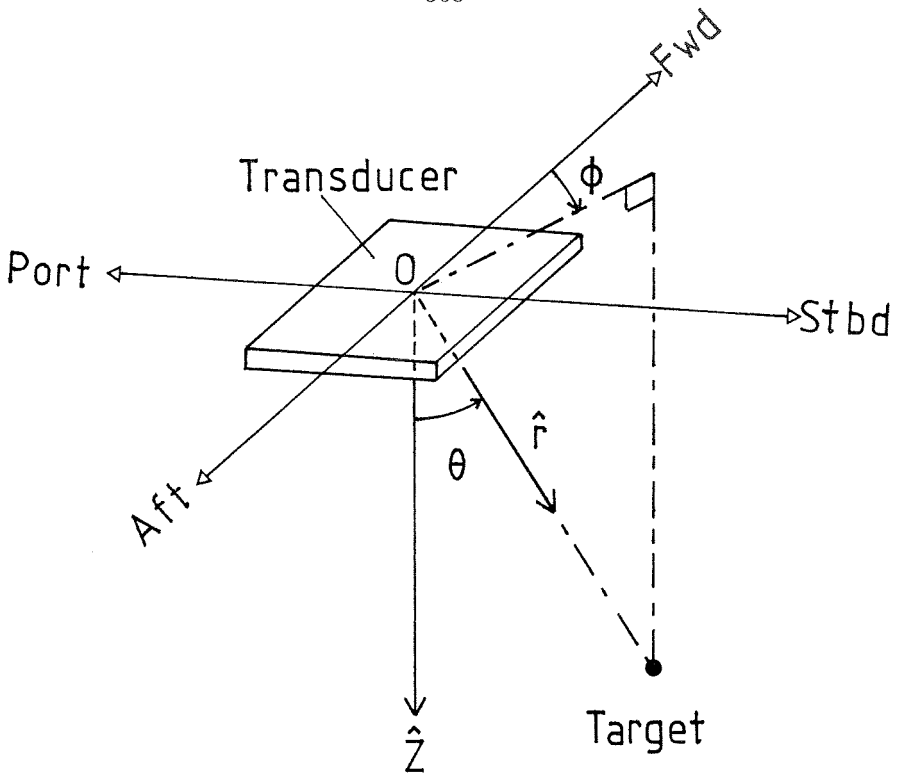


Fig. 2. Geometry of transducer and target. \hat{z} is the acoustic axis of the transducer. The spherical polar coordinates (θ, ϕ) define the target direction \hat{r} relative to the acoustic axis.

If the beam-function compensation were exact, then b would be constant for all \hat{r} within the acceptance cone ($\theta < \theta_m$) and zero otherwise.

EFFECT OF THRESHOLD

The acceptance or rejection of signals is decided *inter alia* by reference to the apparent target direction. The acceptance cone should lie well within the main lobe of the uncompensated beam-pattern, to reduce problems associated with the signal threshold in the receiver.

Suppose that the uncompensated beam-function is B_{\min} at $\theta = \theta_m$ and B_{\max} at $\theta = 0$. Signals below the threshold level are rejected, to discriminate against noise. The cross-section of the smallest detectable target on the acoustic axis is σ_0 . However, this cross-section threshold is increased on the edge of the acceptance cone where targets smaller than $\sigma_1 = (B_{\max}/B_{\min}) \sigma_0$ will be ignored. In effect, the acceptance cone is reduced for very small targets. The problem may be overcome by applying a second threshold to the compensated signal so that all targets for which $\sigma < \sigma_1$ are rejected. The observed

distribution of E has to be truncated at the bottom end. This is an acceptable treatment provided that the dynamic range (signal-to-noise ratio) is sufficiently large.

Suppose now that many targets are detected consecutively by the echosounder. The echoes are separated in time so that they do not overlap, and the signal processor accepts each one as coming from a single target for the purpose of computing the target strength. The unknown probability density function (pdf) of the target cross-sections is denoted $f(\sigma)$. We assume that $f(\sigma)$ and $N\Delta\Omega$, the expected number of targets in solid angle element $\Delta\Omega$ about \hat{r} , are the same for all directions \hat{r} within the acceptance cone.

The pdf of the measured output signals is denoted $g(E)$. It is also convenient to express the variation of the beam function in statistical terms. In particular, $h(b)\Delta b$ is the probability that the beam function for one target is in the range b to $b + \Delta b$ when many targets are isotropically distributed as described above. Thus $h(b)$ is proportional to the solid angle contained by the contours $(b, b + \Delta b)$ of the beam function $b(\hat{r})$.

The functions $g(E)$ and $f(\sigma)$ are related by the following convolution integral (Clay, 1983).

$$g(E) = \int_0^{\infty} h(E/\sigma)[f(\sigma)/\sigma]d\sigma \quad (2)$$

Equation (2) applies for E greater than some threshold E_0 . At or below this, $g(E)$ is zero. The problem now is to deduce the characteristics of $f(\sigma)$ from the observed signal distribution $g(E)$. We begin by considering the statistical moments of the distributions. For positive integer values of n , they are

$$\alpha_n = \int_0^{\infty} E^n g(E)dE \quad (3a)$$

$$\beta_n = \int_0^{\infty} \sigma^n f(\sigma)d\sigma \quad (3b)$$

$$\gamma_n = \int_0^{\infty} b^n h(b)db \quad (3c)$$

Substituting (2) into (3a) and exchanging the order of integration, we find that

$$\begin{aligned} \alpha_n &= \int_0^{\infty} f(\sigma) \left[\int_{E_0}^{\infty} E^n h(E/\sigma) dE/\sigma \right] d\sigma \\ &= \beta_n \gamma_n - \epsilon_n(E_0) \end{aligned} \quad (4)$$

where ϵ_n is an error term due to the non-zero threshold. Reversing the order of integration again and rearranging terms,

$$\epsilon_n = \int_0^{E_0} E^n \left\{ \int_0^\infty h(E/\sigma) [f(\sigma)/\sigma] d\sigma \right\} dE \quad (5)$$

Since $h(b)$ is a compensated beam-function, we would expect it to be significantly non-zero only for a narrow range of b around the mean \bar{b} . If the compensation were exact, G could be described by the Dirac delta function, $h(x) = \delta(x - \bar{b})$. This substitution allows the inner integral in (5) to be evaluated and we derive the approximation

$$\epsilon_n \approx \int_0^{E_0} E^n f(E/\bar{b}) dE/\bar{b} \quad (6a)$$

$$= (\bar{b})^n \int_0^{(E_0/\bar{b})} \sigma^n f(\sigma) d\sigma \quad (6b)$$

Equation (6) shows that ϵ_n depends primarily upon the pdf for targets close to or smaller than the detection threshold. For the consideration of fish targets which produce signals well above the threshold, we may ignore ϵ_n and equation (4) reduces to the simple expression $\alpha_n = \beta_n \gamma_n$.

STATISTICS OF THE TARGET STRENGTH DISTRIBUTION

Although all the moments of $f(\sigma)$ are thus determined explicitly by those of $g(E)$ and $h(b)$, analytic expressions for $f(\sigma)$ in terms of the observed distributions are rather complicated. CLAY (1983) has described a numerical procedure to determine the shape of $f(\sigma)$. For our purposes however, it is sufficient to know the low-order statistics of the distributions, particularly the mean and variance. If the beam-function is non-uniform due to imperfect compensation, we would expect the variance of $g(E)$ to be increased as a result. The significance of the measured beam-function may be considered in terms of the mean and variance of the apparant target strength distribution.

The first moment ($n = 1$) is the mean and so

$$\bar{\sigma} = \bar{E}/\bar{b} \quad (7)$$

The variance depends upon the first and second moments.

$$\begin{aligned} \text{Var}(E) &= \int_0^\infty (E - \bar{E})^2 g(E) dE \\ &= \alpha_2 - \alpha_1^2 \end{aligned} \quad (8)$$

Similar formulae are obtained for $\text{Var}(\sigma)$ and $\text{Var}(b)$. Manipulation of these formulae, and substitutions from equation (4) for $n = 1$ and $n = 2$, leads to the result

$$\begin{aligned} \text{Var}(E)/\bar{E}^2 \\ = \text{Var}(\sigma)/\bar{\sigma}^2 + \text{Var}(b)/\bar{b}^2 + \text{Var}(\sigma)\text{Var}(b)/(\bar{\sigma}\bar{b})^2 \end{aligned} \quad (9)$$

Since the variance cannot be negative, equation (9) demonstrates that the proportion of variance in $g(E)$ is increased by the non-uniformity of the beam-function. The observed target strength distribution is broader than the true distribution by an amount which depends upon the variance of the beam-function in the acceptance cone. The distributions of E and σ are identical only if the beam-function compensation were exact, when $\text{Var}(b)$ would be zero.

CALIBRATION METHOD

The signal from the standard target is measured at two points within the instrument. We require firstly the output signal E which is nominally proportional to the target cross-section, and secondly, the signal V at the transducer terminals or another point before the beam-pattern compensation is applied. The amplitude of V varies with the target position and thus may be used to determine the angle θ of the target relative to the acoustic axis. The procedure for computing θ as a function of V is described below.

For the purpose of estimating the mean sensitivity, the beam cross-section is partitioned into seven equal sub-areas as shown in Figure 3. The acoustic axis is at $\theta = 0$, the centre of the acceptance cone. Targets at angles greater than θ_m are ignored by the instrument. The division of the acceptance cone into the one central and six peripheral sub-areas is particularly convenient when the calibration is to be performed with the target suspended on three lines.

The angular limit of the central sub-area AO is $\theta_c = \theta_m/\sqrt{7}$. The radial lines separating the six segments are equally spaced by 60 degrees in azimuth. The central area is included in addition to the segments because there will usually be several measurements near the acoustic axis. These should be given the same weight as the off-axis measurements.

Suppose that a number of measurements are taken by moving the reference target through all sub-areas of the acceptance cone. Measurements are made at N_i points in area i , namely the sensitivities $b_{i,j}$ and the corresponding angles $\theta_{i,j}$ for $j = 1, 2, \dots, N_i$. The sensitivity is obtained from the measured output signal and the known cross-section of the reference target, using equation (1).

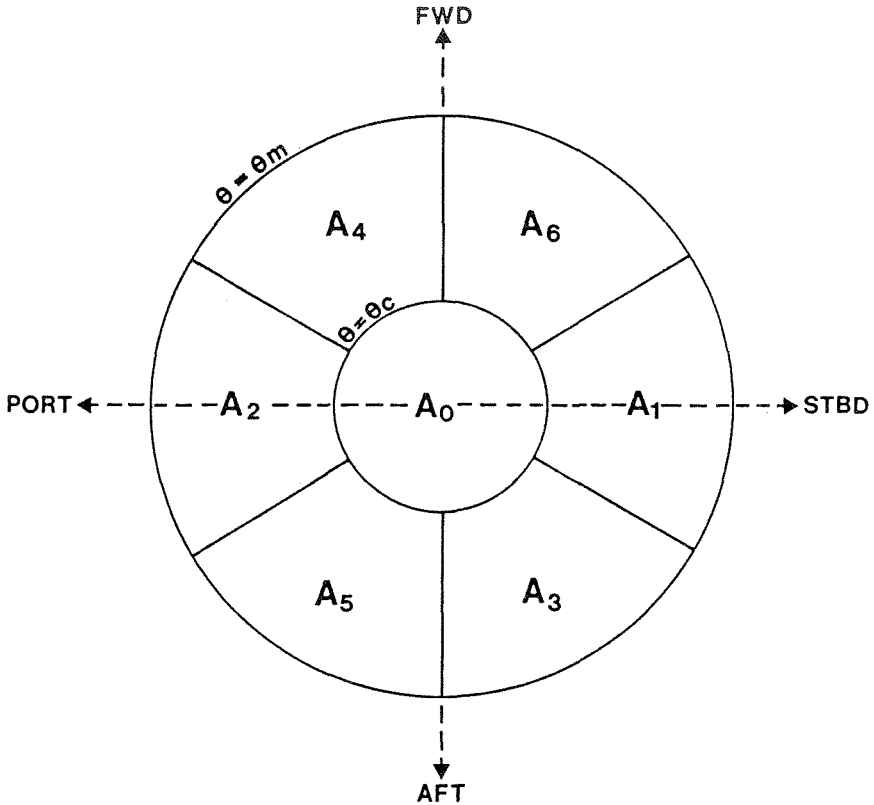


Fig. 3. Divisions of the beam cross-section into seven equal sub-areas. The centre of the diagram is the acoustic axis, at $\theta = 0$. $\theta = \theta_m$ is the acceptance cone. Targets at $\theta > \theta_m$ are ignored.

SAMPLING PROCEDURE

It is preferable that the N_i should all be the same, so that all areas are sampled equally, although some differences in sampling intensity may be accepted with little effect on precision. Ideally, the samples should be located randomly in each area. However, it is expedient to collect the data along radial transects such as the port-starboard line shown in Fig. 3, since the target can be made to move along a transect by adjusting one of the support lines.

To achieve the best coverage of the beam, the samples should be taken at positions such that each sample represents the same area. This implies that the samples should be spaced along the transect in accordance with the following rule.

$$\theta_{i,j} = \sqrt{[(j - \frac{1}{2}) (\theta_m^2 - \theta_c^2)/N_i + \theta_c^2]} \quad (10)$$

The area of the segment between adjacent samples is proportional to the difference in θ^2 . Write $A_i = (\theta_m^2 - \theta_c^2)/N_i$. If the samples are collected in accordance with (10), we have

$$\theta_{i,j+1}^2 - \theta_{i,j}^2 = A_i \quad (11)$$

If it is not possible to position the target precisely, so that the $\theta_{i,j}$ are at arbitrary positions on the transect, each $b_{i,j}$ should be weighted in proportion to the area it represents. Since the cross-section area is proportional to θ^2 , the samples are weighted by elemental areas bounded by the mean θ^2 of adjacent samples, or the inner/outer segment boundary in the case of the first/last samples in a radial transect. The mean sensitivity for segment i is

$$\bar{b}_i = \sum_j b_{i,j} w_{i,j} \quad (12)$$

and the weights $w_{i,j}$ are

$$w_{i,1} = [(\theta_{i,2}^2 + \theta_{i,1}^2)/2 - \theta_c^2]/A_i \quad (13a)$$

$$w_{i,j} = (\theta_{i,j+1}^2 - \theta_{i,j-1}^2)/(2A_i) \text{ for } j = 2, 3, \dots, N_i - 1 \quad (13b)$$

$$w_{i,N_i} = [\theta_m^2 - (\theta_{i,N_i}^2 + \theta_{i,N_i-1}^2)/2]/A_i \quad (13c)$$

If the $\theta_{i,j}$ are those given by (10), the elemental areas are equal and the weights are all unity. The angles of measurements in the central sub-area are ignored and a simple average is calculated there.

$$\bar{b}_o = \sum_i b_{i,o}/N_o \quad (14)$$

Finally, the overall mean sensitivity is

$$\bar{b} = \sum_{j=0}^6 \bar{b}_j/7 \quad (15)$$

For the estimation of the variance, the mean square sensitivity is obtained from an equation similar to (12), substituting b^2 for b .

EXPERIMENTAL METHOD

The equipment required was the same as that used to calibrate conventional echo-sounders, namely a reference sphere (standard target) of known TS and means of suspending the sphere on three lines below the transducer (FOOTE *et al.*, 1987; SIMMONDS *et al.*, 1984). In the case of measurements on the hull-mounted split-beam transducer of «G.O. Sars», the standard target was a 60 mm diameter copper sphere (FOOTE, 1982; MACLENNAN, 1982). A 38.1 mm diameter tungsten carbide sphere was also used for comparison (FOOTE and MACLENNAN, 1984). The on-axis sensitivity measured by the two spheres agreed to within ± 0.1 dB.

The support lines ran from three small winches attached to the sides of the ship, two on the starboard side and one on the port. The sphere was suspended about 18.5 m below the transducer. The length of each line could be adjusted to move the sphere in different directions. Further details of this equipment may be found in FOOTE *et al.* (1987).

An oscilloscope was connected to a suitable point in the echo-sounder to measure the transducer signal V . The line lengths were adjusted until V was at its maximum value, V_0 . The sphere was then on the acoustic axis. The output of the echo-sounder was now recorded to give the first measurement of b in the central area.

The line to the port side winch was now pulled in, causing the sphere to move through sub-area A1 (Fig. 3). At the same time, the amplitude of V was observed. The sphere was moved until V/V_0 corresponds to the angle required for the next measurement.

The angle θ corresponding to the ratio V/V_0 is determined from the uncompensated beam-pattern of the transducer. In the case of the ES400 transducer, the relationship is shown in Fig. 4, which has been compiled from information supplied by the manufacturer.

When the required number of measurements in A1 has been collected, the port side line is released so that the sphere moves to A3 and further measurements are made. Then the sphere is returned to the central sub-area A0 and a measurement may be taken there if desired.

The above procedure is repeated while the other two support lines are adjusted, thus collecting measurements in all the sub-areas A0–A6. The final measurement should be taken in the central area when the sphere is returned to its initial position.

RESULTS AND DISCUSSION

A preliminary investigation of the sensitivity of the ES400 was made on board the research vessel «Dr. Fridtjof Nansen» in August 1985. This indica-

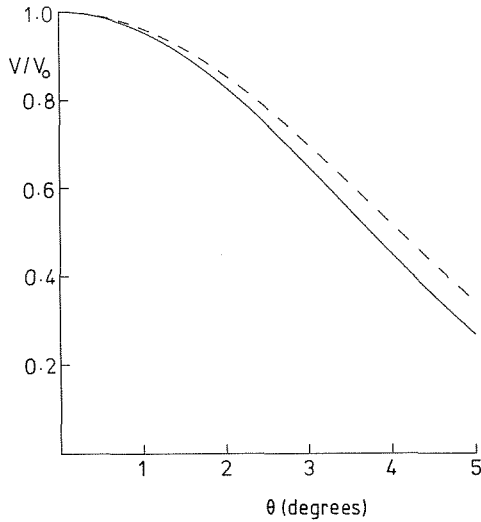


Fig. 4. Beam pattern of the ES400 transducer. The vertical axis is the combined transmit/receive sensitivity relative to 1 on the acoustic axis. Dashed line – sensitivity in the port/starboard plane; solid line – sensitivity in the fore/aft plane.

ted significant changes in sensitivity over the beam. More detailed measurements on another echo-sounder of the same type were obtained during a cruise of «G.O. Sars» in November 1985. Two calibrations were performed. The results from the first calibration are shown in Fig. 5 and summarised in Table 1(a). Each peripheral graph in the figure shows the measurements in one section of the beam corresponding to the adjustment of one support twine. The measurements in the central sub-area A0 were collected at intervals during the calibration.

Following alterations to the echo-sounder, including the installation of a new lobe ROM, the second calibration was performed; the results are shown in Fig. 6 and Table 1(b).

The numerical differences in mean sensitivity between the two calibrations is not important because the gain of the echo-sounder was changed during the adjustments. However, it is seen from Figures 5 and 6 that the variation about the mean sensitivity is similar. In particular, there is a consistent anomaly in area A1 where the sensitivity suddenly increases by more than 2 dB over a small change in θ . We are unable to explain this effect, but note that the ES400 transducer is installed on the port side of the ship, while sub-area A1 is on the starboard side of the beam. It is possible that the keel of the ship, being close to the ray path between the transducer and the targets in A1, might influence the echo-phase to some extent. Whatever the cause, our

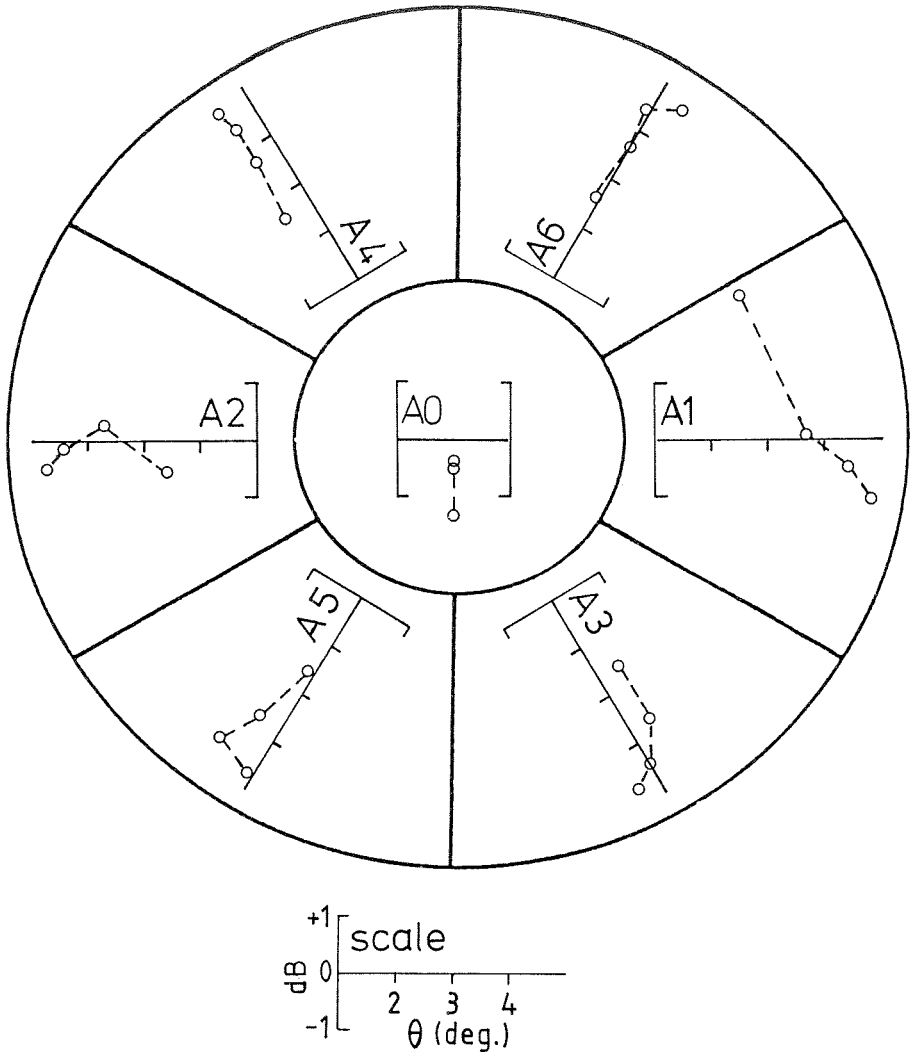


Fig. 5. Results of the first calibration. The measured sensitivity 'b' is shown as a function of the target position in the beam. Each graph shows the measurements in one sub-area. Radial axes – target direction relative to the acoustic axis (θ); azimuthal axes – measured sensitivity in decibels relative to the mean sensitivity for that sub-area.

results demonstrate that significant changes can occur in the sensitivity of the split-beam echo-sounder, emphasising the need for appropriate calibration procedures to correct the observed target strengths.

We observed more random variation during the second calibration. This is reflected in the wider confidence limits, ± 0.6 dB as opposed to ± 0.4 dB on the mean sensitivity, and the proportional variance is three times larger.

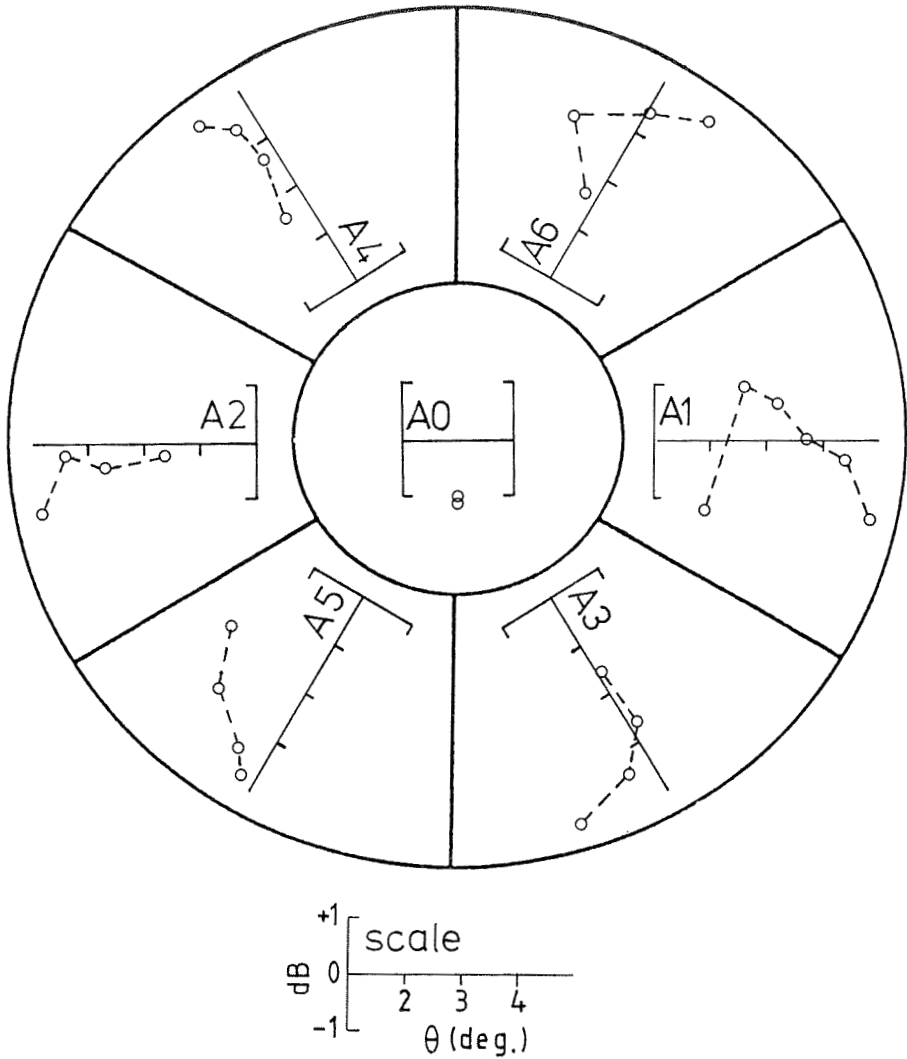


Fig. 6. Results of the second calibration. The measured sensitivity 'b' is shown as a function of the target position in the beam. Each graph shows the measurements in one sub-area. Radial axes - target direction relative to the acoustic axis (θ); azimuthal axes - measured sensitivity in decibels relative to the mean sensitivity for that sub-area.

Some of this variation will be measurement error, arising for example when fish swim close to the target. Interference from fish is revealed by fluctuation of the echo level. Should this happen, it is best to postpone the calibration; wait until the fish go away, or find another place to do the measurements.

However, the difference between the two calibrations might indicate real changes in sensitivity, since the sphere would be unlikely to have moved along

Table 1

Results from two calibrations of the ES400 echo-sounder. The receiver gain was adjusted between the calibrations. \bar{b}_i is the mean sensitivity per sub-area as defined in the text. \bar{b} is the overall mean in the acceptance cone which has been estimated within 95 % confidence limits. The variance of the sensitivity is a measure of the uniformity of the beam-pattern compensation.

Sub-area	(a) First calibration		(b) Second calibration	
	No of points	Sensitivity (\bar{b}_i)	No of points	Sensitivity (\bar{b}_i)
A0	3	0.930	2	2.289
A1	4	1.342	6	2.863
A2	4	1.054	4	2.676
A3	4	1.066	4	2.748
A4	4	0.980	4	2.644
A5	4	1.203	4	3.808
A6	4	1.095	4	3.121
A0-A6	(\bar{b}) 27	(\bar{b}) 1.10 \pm 0.10	28	2.88 \pm 0.44
95 % limits on the observed TS		\pm 0.4 dB		\pm 0.6 dB
Var (b)/(\bar{b})		0.056		0.163

the same transects in the two cases. More detailed measurements would be necessary to investigate this possibility.

Fish target strength is a highly variable parameter. In typical applications, $\text{Var}(\sigma)/\bar{\sigma}^2$ would be expected to be much larger than the worse case 16 % that we found for $\text{Var}(b)/\bar{b}^2$. Thus the non-uniformity of the beam should have little effect on the distribution of the observed target strengths, but a correction may be applied through equation (9) if required.

CONCLUSIONS

The split-beam echo-sounder may be calibrated using the same equipment as for conventional echo-sounders. Measurements are required of the echo from a standard target at various positions in the transducer beam. Given about 30 measurements, the mean sensitivity can be determined to better than ± 0.5 dB at the 95 % confidence level. The calibration will be more accurate if more measurements are made.

The technique has been applied to the calibration of the ES400 echo-sounder. We observed substantial variation of the sensitivity within the acceptance cone, up to 2 dB. The cause of this variation is not known. However, it demonstrates the need for careful calibration of the instrument, in-

cluding sensitivity measurements at many positions in the acceptance cone, before the split-beam echo-sounder may be used to produce reliable target strength data. Measurements at many positions are necessary to detect anomalous changes in the sensitivity. It is insufficient to rely upon one measurement of a target on the acoustic axis. Furthermore, it is necessary to determine the variance as well as the mean of the directional sensitivity, in order to quantify the effect of non-uniformity on the observed target strength distribution.

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