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# THEORETICAL ESTIMATION OF THE MEAN ECHO INTENSITY — FISH NUMBER DENSITY RELATION FOR ENCAGED SAITHE IN THE DORSAL ASPECT

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## ABSTRACT

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The formulation of an earlier theoretical study on the scattering of sound by encaged aggregations of fish (FOOTE 1978) is applied to the problem of the scattering of sound by an encaged aggregation of saithe when ensonified dorsally. The relationship of the mean time-integrated echo intensity  $\overline{e}$  and fish number density  $\nu$  is computed for narrow-band pulsed sinusoidal signals of center frequency 38 kHz and 120 kHz for the same constant geometric and corresponding physical quantities as in the earlier study, but with use of the pertinent dorsal aspect target strength data in place of ventral aspect data. The computed  $\overline{e}$ - $\nu$  relationships are compared with the corresponding ventral aspect results as obtained experimentally by RØTTINGEN (1976) and modelled theoretically in FOOTE (1978).

## INTRODUCTION

In a recent paper (FOOTE 1978) some experimental results on the scattering of ultrasonic sound by encaged aggregations of fish were analyzed. The subject of the analysis, the empirical results of RØTTINGEN (1976), was expressed in the form of relationships of the normalized, mean time-integrated echo intensity  $\bar{\epsilon}$  to fish number density  $\nu$ . These relationships had been obtained for a range of conditions of ensonification by a pulsed sinusoidal signal for two different kinds of fish, *Pollachius virens* (L.) or saithe, and *Sprattus sprattus* (L.) or sprat. The insensitivity in the forms of the empirical  $\bar{\epsilon}$ - $\nu$  relationship to both the pulse duration and center frequency of the ensonifying signal for each species of fish was explained qualitatively by purely geometric considerations. A theory was then constructed and applied to the case of the saithe, for which more pertinent scattering data were available than for the case of the sprat.



Fig. 1. Dorsal aspect target strength of saithe of mean length  $35.1 \pm 0.6$  cm at 38 kHz when averaged with respect to 16 specimens, and at 120 kHz when averaged with respect to 17 specimens.

The fish in Røttingen's experiments were ensonfied ventrally in each case, as the transducers were located in a fixed position below the net cage. The corresponding ventral aspect target strength data were used in the quantitative analysis. Because of interest in at-sea applications of this work in fisheries research, where fish are generally ensonified dorsally, an evaluation of the theoretical  $\overline{\epsilon}$ - $\nu$  relationship for saithe in the dorsal aspect is presented here for the same conditions of ensonification that obtained during the experiments.

#### METHOD

The evaluation of the theoretical expression for acoustic scattering by an aggregation of fish, as developed and applied in FOOTE (1978) to the particular circumstances of Røttingen's experiment, proceeds similarly to

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Fig. 2. Mean backscattering cross section  $\overline{\sigma}_{b}$  of saithe in dorsal aspect at 38 kHz as a function of the spread  $\sigma_{a}$  in tilt angle distribution with mean tilt angle  $\overline{\theta}$  as a parameter.

that of the quoted study. The only difference is that the pertinent dorsal aspect target strength data are used in place of the ventral aspect target strength data.

The new dorsal aspect target strength data for saithe of mean length  $35.1 \pm 0.6$  cm are shown in Fig. 1. The corresponding ensemble-averaged backscattering cross sections are presented in Fig. 2 and 3 for the respective cases of 38 kHz and 120 kHz. These latter quantities were computed with respect to the identical parameters of the orientation distribution of the earlier study.

#### RESULTS

The results of the evaluation of the model are shown in Fig. 4 and 5 for the respective cases of 38 kHz and 120 kHz signals. In both figures the same geometric conditions that obtained in FOOTE (1977) apply. In particular, it was assumed that the mean tilt angle  $\theta$  of the orientation



Fig. 3. Mean backscattering cross section  $\overline{\sigma}_b$  of saithe in dorsal aspect at 120 kHz as a function of the spread  $\sigma_{\theta}$  in tilt angle distribution with mean tilt angle  $\overline{\theta}$  as a parameter.

distribution was 0 degrees and that the mean spread  $\sigma_{\theta}$  in tilt angle distribution has the following form:

$$\sigma_{\theta} = [\sigma_{\theta,1,0}^2 \exp(-2v/v_{\rm cr}) + 4.2]^{1/2}$$

where  $\sigma_{\theta,1,0}$ , which is the free-space spread in tilt angle distribution, is 18 degrees, and  $v_{\rm cr}$ , or critical fish number density, is either 100 or 125 fish/m<sup>3</sup>. The same extinction cross sections that were used earlier were used here; namely,  $\sigma_{\rm e,o} = 60 \text{ cm}^2$  for the 38 kHz signal and  $\sigma_{\rm e,o} = 100 \text{ cm}^2$  for the 120 kHz signal.

### DISCUSSION

The predicted normalized  $\overline{\epsilon}$ - $\nu$  relationships for saithe in the dorsal aspect, cf. Fig. 4 and 5, are rather different from the corresponding relationships for the ventral aspect as observed by RØTTINGEN (1976) and



Fig. 4. Theoretical normalized mean time-integrated echo intensity  $\overline{\epsilon}$  as a function of density  $\nu$  for saithe (in the dorsal aspect) at 38 kHz for the following model parameters:  $\overline{\theta} = 0$  degrees;  $\sigma_{e, \theta} = 60$  cm<sup>2</sup>;  $\sigma_{\theta, 1, 0} = 18$  degrees;  $\nu_{cr} = 100$  and 125 fish/m<sup>3</sup>.

reproduced theoretically by FOOTE (1977). The source of these differences in the case that the corresponding ventral and dorsal aspect extinction cross sections are identical and all geometric quantities remain unchanged, according to theory, lies entirely in differences in the tilt angle dependences of corresponding ventral and dorsal aspect target strength data, thence to differences in the *p*-dependences of the corresponding ensemble-averaged backscattering cross sections. These differences are illustrated in Fig. 6 and 7, which show the dependence of  $\overline{\sigma}_{b}$  on the spread  $\sigma_{\theta}$  in tilt angle distribution for a mean inclination of 0 degrees



Fig. 5. Theoretical normalized mean time-integrated echo intensity  $\overline{e}$  as a function of density v for saithe (in the dorsal aspect) at 120 kHz for the following model parameters:  $\overline{\theta} = 0$  degrees;  $\sigma_{e,o} = 100 \text{ cm}^2$ ;  $\sigma_{\theta,1,0} = 18$  degrees;  $v_{cr} = 100$  and 125 fish/m<sup>3</sup>.



Fig. 6. Mean backscattering cross sections of saithe in dorsal and ventral aspects at 38 kHz as a function of the spread  $\sigma_{\theta}$  in tilt angle distribution for mean tilt angle of 0 degreses.

for signals with respective center frequencies of 38 kHz and 120 kHz. The dorsal aspect curves shown here were extracted from Fig. 2 and 3, while the ventral aspect curves were taken from Fig. 5 and 6 of FOOTE (1978).

In the case of the 38 kHz signal the difference in behaviour of  $\bar{\sigma}_{\rm b}$  for  $\sigma_{\theta}$  less than 5.5 degrees, cf. Fig. 6, is significant. For these values of  $\sigma_{\theta}$ , which correspond to values of  $\nu$  greater than about 125 fish/m<sup>3</sup>,  $\bar{\sigma}_{\rm b}$  decreases with decreasing  $\sigma_{\theta}$  or, equivalently, with increasing  $\nu$  for the ventral aspect, while remaining nearly constant for the dorsal aspect. The corresponding  $\bar{\epsilon}$ - $\nu$  relationships show a definite peaking for  $\nu$  approximately equal to 160 fish/m<sup>3</sup> in the ventral aspect, and a steady monotonic increase to the maximum computed density, 350 fish/m<sup>3</sup>, in the dorsal aspect.

The dependences of  $\bar{\sigma}_b$  on  $\sigma_{\theta}$  for the case of a 120 kHz signal are very similar at all values of  $\sigma_{\theta}$  in excess of about 2 degrees. Because the irreducible perspectival contribution to  $\sigma_{\theta}$  exceeds 2 degrees for the particular conditions of Røttingen's experiment, the dependences of  $\bar{\sigma}_b$  on v over the entire range of values of v can be considered similar for both aspects. Thus the  $\bar{\epsilon}$ -v relationships, after appropriate normalization, are seen to be nearly identical at 120 kHz.

An assumption which has been applied consistently in the computations of this paper is that the dorsal and ventral aspect extinction cross sections are identical. Thus the dorsal aspect extinction cross section has been equated to 60 cm<sup>2</sup> and 100 cm<sup>2</sup> for the respective cases of 38 kHz and 120 kHz signals. It is noted that there is no a priori reason why the dorsal



Fig. 7. Means backscattering cross sections of saithe in dorsal and ventral aspects at 120 kHz as a function of the spread  $\sigma_{\theta}$  in tilt angle distribution for mean tilt angle of 0 degreses.

and ventral aspect cross sections should be equal, except in the limit of sufficiently high frequencies. In this limit the pheomenon of scattering is said to become geometric, and the total scattering cross section, thence extinction cross section in the case that absorption within the fish is negligible, is equal to twice the net projected area of the fish. Thus the total scattering cross sections of the fish in ventral and dorsal aspects, and, presumably, the ventral and dorsal aspect extinction cross sections, would be equal.

With respect to the specimens of saithe and signal frequencies of Røttingen's experiment it was found that the characteristic scattering sizeto-wavelength ratio was generally much greater than unity, i.e., that the ensonifying frequencies were high. However, the high frequency asymptotic limit evidently did not obtain, for the extinction cross section was observed to depend on frequency. In this case, in which scattering is nongeometric, or frequency dependent, the dorsal and ventral aspect extinction cross sections, like their backscattering cross sections, may be different. The effect of a change in the extinction cross section on the normalized  $\overline{e}$ - $\nu$  relationships of Fig. 4 and 5 will be similar to that shown in Fig. 10 of FOOTE (1977) by virtue of the mathematical equivalence of the quantitative descriptions of ventral and dorsal aspect scattering.

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