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RELATION OF MAXIMUM AND AVERAGE TARGET STRENGTHS FOR THE SAME FISH

by

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ABSTRACT

The relation is studied by means of tabulated measurements of the tilt angle dependence of target strength for three gadoid species. The dependence of the difference target strength on fish length is statistically investigated for a range of simulated behaviour modes.

INTRODUCTION

Imagine a situation where multiple observations can be made on the same fish with an echo sounder. If the target strength (TS) can be directly measured at the same time, then the observed fish can be characterized by a set of TS samples. In order to use this data set together with other data sets, it is desirable to compress the set to a much smaller set, ideally consisting of a single number. But what should this number be? If either the maximum or average TS is chosen, is the other quantity implied? That is, can it be determined by some *a priori* relation?

The purpose of this contribution is to investigate the relation of maximum and average TS for the same fish.

MATERIAL AND METHODS

In order to systematize any possible relation of maximum and average TS, it is important that the object fish, namely species, size and orientation, be known precisely. Thus reference is made to a proven set of measurements of the TS functions of fish, in this case of gadoids at 38 kHz by Nakken and Olsen (1977). Specifically, the data consist of TS functions of tilt angle of 68 cod (*Gadus morhua*), 59 saithe (*Pollachius virens*), and 44 pollack (*Pollachius pollachius*) (Foote and Nakken 1978).

Extraction of the maximum TS of each function is trivial. Regression analyses of the quantity on total fish length have been published in several places, e.g., in Nakken and Olsen (1977) and Foote (1979).

Computation of the average TS is a straightforward procedure. First the TS function of tilt angle θ is expressed through the backscattering cross section σ by the usual definition,

$$TS = 10 \log \frac{\sigma}{4\pi} \quad , \tag{1}$$

which follows Urick (1975), except with use of SI units. The average backscattering cross section is now computed. For a given normal distribution $N(\overline{\theta},s_\theta)$ and uniform distribution of fish in the horizontal plane over a conical section, the effect of perspective is introduced through use of an effective standard deviation $s_\theta' \gtrsim s_\theta$ in $f(\theta)$ in the expression

$$\overline{\sigma} = \int_{\overline{\theta}-3s_\theta}^{\overline{\theta}+3s_\theta} \sigma(\theta)\ f(\theta)\ d\theta \quad , \tag{2}$$

where the normalization of $f(\theta)$ reflects truncation of the θ-range at $\overline{\theta}\pm 3s_\theta$. The advantage of the approximation is that the threefold integration in Foote (1980) is reduced to a single integration, as in Foote (1985). The average TS is, by definition, the quantity resulting from substituting $\overline{\sigma}$ in equation (2) into equation (1).

Systematizing the relation of maximum and average TS consists in regressing the difference $TS_{max}-\overline{TS}$ on fish length for each of a set of simulated behaviour modes for each gadoid species. The behaviour modes are defined through the normal distribution with mean $\overline{\theta} \in \{-30,-25,...,30\}$ deg for each of three standard deviations $s_\theta \in \{5,10,15\}$ deg. The object fish are assumed to occur anywhere in a horizontal cross section of an ideal conical beam with 10 deg vertex angle, representing the ideal beam of a typical downwards-looking transducer used in echo surveying of fish stocks, with equally likely probability. Thus the effective standard deviation to be used in $f(\theta)$ in equation (2) is $s_\theta' = \{5.5,10.2,15\}$ deg.

The particular form of the regression equation is the following:

$$TS_{max} - \overline{TS} = m \log \ell + b \quad , \tag{3}$$

where ℓ is the total fish length in centimeters.

To determine whether the length dependence is significant, a t-test has been performed on the quantity

$$t = \frac{m}{SE_m} \quad , \tag{4}$$

where SE_m is the standard error of m. The value is compared with the t-statistic $t_{\alpha(2),(n-2)}$, where α denotes the level of significance, and n is the number of basis TS functions in the regression analysis. For $\alpha=0.05$, and the particular values of n stated earlier, $t_{0.05(2),66}=1.997$, $t_{0.05(2),57}=2.002$, and $t_{0.05(2),42}=2.018$.

Notwithstanding the significance of the length dependence, regression analyses for the same behaviour mode but different species have been examined for coincidences of slopes m, and, in the event, for coincidence of

Table 1. Regression analyses of $TS_{max}-\overline{TS}$ on fish length for 68 cod. Total length range is 6.7-96.0 cm.

$\overline{\theta}$ (deg)	s_θ (deg)	$TS_{max}-\overline{TS} = m \log \ell + b$: m	b	SE	$TS_{max}-\overline{TS} = b$: b	SE	t
-30	5	1.8	11.2	3.6	13.0	3.4	1.45
-25	5	3.1	7.0	3.4	11.7	3.5	4.97
-20	5	3.5	3.4	3.3	8.8	3.4	7.15
-15	5	3.1	1.0	2.6	5.8	2.8	8.74
-10	5	2.8	-0.5	1.7	3.7	1.0	15.50
-5	5	3.1	-1.7	1.0	3.0	1.4	61.50
0	5	4.2	-2.5	1.4	3.8	1.9	55.79
5	5	5.0	-1.8	2.0	5.8	2.5	38.43
10	5	4.9	0.6	2.6	8.1	3.0	21.59
15	5	3.9	4.3	3.1	10.2	3.3	9.19
20	5	1.7	9.3	3.3	11.9	3.3	1.64
25	5	-0.5	14.2	3.7	13.5	3.7	0.10
30	5	-1.4	17.2	4.5	15.1	4.5	0.57
-30	10	2.6	7.4	2.7	11.3	2.8	5.50
-25	10	2.8	4.8	2.3	9.1	2.4	8.84
-20	10	2.9	2.7	1.9	7.1	2.1	14.03
-15	10	2.9	1.1	1.5	5.5	1.7	24.68
-10	10	3.1	-0.1	1.1	4.6	1.4	50.49
-5	10	3.3	-0.8	0.8	4.2	1.3	99.09
0	10	3.6	-1.0	0.9	4.5	1.4	109.37
5	10	3.9	-0.6	1.1	5.4	1.6	75.89
10	10	4.1	0.5	1.5	6.7	1.9	48.04
15	10	4.0	2.3	1.8	8.4	2.2	29.00
20	10	3.5	4.8	2.2	10.1	2.4	15.81
25	10	2.7	7.8	2.5	11.9	2.6	6.87
30	10	1.7	11.0	3.0	13.5	3.0	1.96
-30	15	2.7	5.4	1.9	9.5	2.0	12.56
-25	15	2.8	3.8	1.6	8.1	1.8	18.81
-20	15	2.9	2.4	1.4	6.9	1.6	28.39
-15	15	3.1	1.3	1.1	6.0	1.5	43.56
-10	15	3.2	0.6	1.0	5.5	1.4	64.31
-5	15	3.3	0.2	0.9	5.3	1.3	87.08
0	15	3.5	0.1	0.9	5.4	1.4	96.37
5	15	3.6	0.4	0.9	5.9	1.4	88.72
10	15	3.6	1.1	1.1	6.6	1.5	69.25
15	15	3.7	2.1	1.3	7.6	1.7	50.00
20	15	3.6	3.4	1.5	8.9	1.8	35.15
25	15	3.4	5.1	1.7	10.3	2.0	23.13
30	15	3.1	7.1	2.0	11.8	2.2	14.35

Table 2. Regression analyses of $TS_{max}-\overline{TS}$ on fish length for 59 saithe. Total length range is 9.1-68.0 cm.

$\overline{\theta}$	s_θ	$TS_{max}-\overline{TS} = m \log \ell + b$			$TS_{max}-\overline{TS} = b$		
(deg)	(deg)	m	b	SE	b	SE	t
-30	5	-4.6	23.1	4.5	16.5	4.6	4.37
-25	5	0.1	14.8	3.7	15.0	3.7	0.00
-20	5	2.6	8.1	3.0	11.9	3.1	3.23
-15	5	3.0	3.6	2.2	7.9	2.3	7.62
-10	5	2.7	0.8	1.5	4.7	1.6	14.45
-5	5	3.1	-1.3	0.8	3.2	1.2	60.49
0	5	4.2	-2.6	0.8	3.4	1.4	118.42
5	5	4.7	-1.6	1.6	5.2	2.0	37.93
10	5	2.7	3.7	2.3	7.6	2.4	5.78
15	5	-2.2	13.2	2.5	10.0	2.6	3.25
20	5	-6.5	21.7	2.8	12.2	3.3	22.87
25	5	-7.0	23.9	3.2	13.8	3.7	19.73
30	5	-5.8	23.8	3.8	15.4	4.0	10.29
-30	10	-0.1	13.5	2.4	13.4	2.3	0.01
-25	10	1.6	8.7	1.9	11.0	1.9	2.90
-20	10	2.4	5.1	1.6	8.6	1.7	10.50
-15	10	2.9	2.5	1.2	6.7	1.4	23.11
-10	10	3.2	0.8	1.0	5.3	1.3	45.77
-5	10	3.4	-0.2	0.8	4.6	1.2	78.92
0	10	3.5	-0.4	0.8	4.6	1.2	85.18
5	10	3.3	0.5	1.0	5.3	1.3	50.27
10	10	2.7	2.6	1.3	6.5	1.5	19.75
15	10	1.5	5.9	1.6	8.1	1.6	3.82
20	10	-0.3	10.5	1.9	10.0	1.9	0.14
25	10	-2.4	15.5	2.3	12.0	2.4	4.78
30	10	-4.1	20.1	2.8	14.1	3.0	9.54
-30	15	1.7	8.6	1.5	11.0	1.6	5.07
-25	15	2.2	6.1	1.3	9.3	1.4	12.43
-20	15	2.6	4.1	1.2	7.9	1.3	22.58
-15	15	2.9	2.7	1.0	6.9	1.3	34.87
-10	15	3.0	1.7	0.9	6.1	1.2	47.36
-5	15	3.1	1.2	0.9	5.7	1.2	55.08
0	15	3.0	1.3	0.9	5.7	1.2	53.23
5	15	2.9	1.8	0.9	6.0	1.2	41.61
10	15	2.6	3.0	1.0	6.7	1.2	26.11
15	15	2.1	4.6	1.2	7.6	1.3	13.27
20	15	1.4	6.8	1.4	8.8	1.4	4.43
25	15	0.6	9.5	1.6	10.3	1.6	0.54
30	15	-0.4	[illegible]	1.9	[illegible]	1.8	[illegible]

Table 3. Regression analyses of $TS_{max}-\overline{TS}$ on fish length for 44 pollack. Total length range is 19.7-61.0 cm.

$\overline{\theta}$ (deg)	s_θ (deg)	$TS_{max}-\overline{TS} = m \log \ell + b$: m	b	SE	$TS_{max}-\overline{TS} = b$: b	SE	t
-30	5	7.0	4.5	4.1	14.6	4.2	1.96
-25	5	8.3	1.9	4.2	13.9	4.3	2.65
-20	5	6.5	2.2	3.5	11.7	3.6	2.33
-15	5	4.3	1.5	2.4	7.8	2.4	2.27
-10	5	3.5	-0.5	1.6	4.6	1.6	3.32
-5	5	3.4	-1.9	1.0	3.1	1.0	8.54
0	5	3.7	-1.9	0.6	3.5	0.8	22.34
5	5	3.4	0.8	1.2	5.7	1.3	5.63
10	5	2.9	4.7	2.5	8.9	2.5	0.89
15	5	4.9	4.2	4.2	11.3	4.2	0.93
20	5	6.8	2.9	5.3	12.9	5.3	1.15
25	5	8.1	2.6	6.1	14.4	6.1	1.20
30	5	10.3	1.2	7.2	16.2	7.2	1.42
-30	10	5.4	5.0	2.9	12.9	3.0	2.32
-25	10	5.1	3.3	2.3	10.6	2.4	3.23
-20	10	4.5	1.9	1.8	8.4	1.8	4.28
-15	10	4.0	0.7	1.4	6.6	1.5	5.70
-10	10	3.7	-0.1	1.1	5.3	1.2	7.75
-5	10	3.5	-0.4	0.9	4.7	1.0	10.80
0	10	3.3	-0.1	0.8	4.8	0.9	12.40
5	10	3.2	0.8	0.9	5.6	1.0	8.31
10	10	3.4	2.0	1.4	6.9	1.4	4.14
15	10	4.0	2.9	2.1	8.7	2.1	2.48
20	10	5.2	3.1	2.9	10.6	3.0	2.13
25	10	7.0	2.4	4.0	12.6	4.0	2.14
30	10	8.6	2.1	5.1	14.6	5.1	1.96
-30	15	4.5	4.2	1.9	10.7	1.9	3.90
-25	15	4.3	2.0	1.6	9.1	1.7	5.02
-20	15	4.1	1.9	1.4	7.8	1.4	6.13
-15	15	3.9	1.1	1.2	6.8	1.3	7.39
-10	15	3.7	0.7	1.1	6.1	1.1	8.37
-5	15	3.6	0.5	1.0	5.8	1.1	9.36
0	15	3.5	0.7	1.0	5.8	1.0	8.99
5	15	3.5	1.1	1.0	6.2	1.1	7.73
10	15	3.6	1.6	1.2	6.9	1.3	5.86
15	15	3.9	2.2	1.5	7.9	1.6	4.41
20	15	4.4	2.7	1.9	9.2	2.0	3.56
25	15	5.2	3.1	2.4	10.6	2.5	3.14
30	15	6.0	3.4	2.9	12.2	3.0	2.88

Table 4. Regression analyses of $TS_{max}-\overline{TS}$ on fish length for 171 merged gadoids together with F-values for testing for coincidences among the slopes m and intercepts b of respective constituent regression analyses in Tables 1-3.

$\overline{\theta}$ (deg)	s_θ (deg)	$TS_{max}-\overline{TS} = m \log \ell + b$: m	b	SE	F_m	F_b
-30	5	-0.9	16.3	4.3	4.21	6.22
-25	5	1.4	11.3	4.0	1.75	13.47
-20	5	2.5	7.0	3.6	0.44	20.28
-15	5	2.5	3.3	2.6	0.09	18.18
-10	5	2.5	0.6	1.7	0.08	10.18
-5	5	3.0	-1.4	0.9	0.04	2.45
0	5	4.2	-2.5	1.0	0.07	0.13
5	5	4.8	-1.5	1.7	0.28	1.19
10	5	3.9	2.4	2.5	1.09	3.49
15	5	1.5	8.3	3.4	4.72	2.26
20	5	-1.3	14.2	3.9	7.66	0.74
25	5	-2.5	17.6	4.4	5.02	0.40
30	5	-2.5	19.2	5.1	3.23	0.54
-30	10	1.1	10.8	2.8	1.96	12.62
-25	10	1.9	7.3	2.4	0.89	16.71
-20	10	2.3	4.5	1.9	0.40	18.73
-15	10	2.6	2.3	1.5	0.21	18.64
-10	10	2.9	0.7	1.1	0.11	16.84
-5	10	3.2	-0.2	0.9	0.02	12.45
0	10	3.4	-0.5	0.8	0.08	5.50
5	10	3.6	0.1	1.0	0.52	2.08
10	10	3.5	1.6	1.4	1.23	1.50
15	10	3.0	4.0	1.8	2.40	1.39
20	10	2.0	7.2	2.4	4.09	1.21
25	10	0.8	10.9	3.0	5.51	1.05
30	10	-0.3	14.5	3.7	5.54	1.13
-30	15	2.0	7.4	1.9	0.95	15.55
-25	15	2.3	5.4	1.6	0.61	17.32
-20	15	2.6	3.7	1.4	0.38	18.25
-15	15	2.8	2.4	1.2	0.24	18.14
-10	15	2.9	1.5	1.1	0.14	16.51
-5	15	3.1	1.0	1.0	0.14	13.58
0	15	3.1	1.0	0.9	0.30	9.14
5	15	3.2	1.3	1.0	0.65	5.44
10	15	3.1	2.1	1.1	1.25	2.95
15	15	3.0	3.3	1.3	1.90	1.80
20	15	2.7	4.9	1.6	2.72	1.20
25	15	2.3	6.9	1.0	3.56	0.97
30	15	1.8	9.3	2.3	4.32	1.01

intercepts b. This is done by analysis of covariance, as in Zar (1974). The derived measures are tested against the F-statistics: $F_{0.05(1),2,166}=3.05$ for slopes, and $F_{0.05(1),2,167}=3.05$ for intercepts.

RESULTS

These are presented in Tables 1-3 for the respective species. The results of regression analyses with the merged gadoid data are presented in Table 4 together with F-values for testing for coincidences among the three constituent data sets.

DISCUSSION

Use of t-testing on the gadoid data indicates that the slope or length dependent coefficient in equation (3) is for the most part non-vanishing. The exceptions generally occur for rather extreme behaviour modes, with mean tilt angles far from the horizontal. For more ordinary behaviour, the coefficient is very roughly 3-4. This suggests the the magnitude of the increase in maximum TS over the average TS with increasing fish length.

Application of covariance analysis indicates a general similarity or coincidence of the regression slopes for corresponding behaviour modes. However, the intercepts are generally significantly different, so merging of the data with respect to species is contraindicated.

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