ON THE DETERMINATION

OF

THE SALINITY OF SEA WATER

BY ITS

POWERS OF REFRACTION

BY

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n a previous occasion I formed a method of determining the salinity of sea water by the aid of its electrical conductivity*. The object of this method was to obtain the greatest possible exactness, even when working on board a ship at sea. Electrical conductive power was, at that time (the work was carried out in 1893) the only physical quantity, by the measurement of which any hope might be entertained of gaining greater accuracy in the results that had, hitherto, been obtained.

The method, undoubtedly, gives very considerable exactitude, but, nevertheless, suffers from the fault that, the calculation of the results craves considerably more time than is desirable, and that this failing cannot be overcome by forming subsidiary tables, of any reasonable compass, in order to carry out the calculations.

The method is, thus, not suitable for employment in cases where it is necessary to make the greatest possible number of decisions in a short time, and when, besides, it is desirable to calculate the results on the spot.

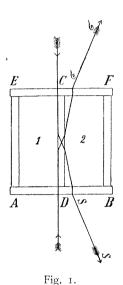
After that W. Hallwachs had, subsequently, published a method**, by which the relative refractory powers of two liquids of slightly different optical density may be measured with far greater accuracy than was, formerly, possible, I deemed it probable that, by measuring the refractory power of salt water, an increase of exactitude might be gained in determining the salinity, this being a thing so greatly needed in the study of hydrography. The salinity, as measured by refraction, also demands considerable time in calculating the results, but, in this instance, the time may be reduced to a minimum by the preparation of subsidiary tables of small compass. When I had an opportunity of explaining my views on this point to Dr. Hjort, who during a series of years has conducted the investigations in this country, I was requested by him to work out the method in detail, and, besides, to see if it could be suitably

^{*)} Nyt Magazin for Naturvidenskaberne 34-232.

^{**)} Wiedemanns Annalen 50--577.

employed. In April 1896, I, therefore, made a series of investigations of this subject, which I shall deal with below.

The Hallwach differential prism is formed of a rectangular parallelopipedic glass vessel, divided into two portions by a partition. Fig. 1, shews the prism as seen from above. According to Hallwach's statement it is sufficient if the plates AB and CD are on perpendicular parallel glasses. The two half portions of the vessel, nos. 1 and 2 are to be filled respectively with pure water and sea water.



The rays which, in pure water, would fall perpendicularly on plate AB, and thereby glance along the partition CD, are clearly transmitted to the sea water, and leave the prism as parallel rays which form a decided angle (α) with the partition. It is easy to see that all the rays, which, otherwise, enter the partition at an angle of incidence less than 90°, will leave the prism in directions which, with the partition, form angles greater than α . When the broken luminous fascicle is seen through the refractometer telescope, it is clear that the rays which, in the field of vision, correspond to the smallest angle α , will appear as a sharp straight line, parallel to the partition, forming the boundary of the luminous fascicle.

The double angle 2 α is to be measured according to Hallwach's method, by first adjusting the reticule on the said straight lined boundary of the broken luminous fascicle, and then, after having changed sea water for pure water, and pure water for sea water, repeating the adjusting process.

In order to avoid changing the sea water and pure water, I have had the prisms arranged so that both the plates AB and EF are made of parallel glasses, and that both of these are fixed perpendicularly to the plano-parallel partition. The prism is thus quite symetrical, and it matters not through which of the plates AB and EF the light enters or leaves the prism. When the light enters at C, with glancing incidence, it will clearly leave the prism in the direction s s. On the other hand, if it enters at D, it will leave the prism in the direction $\sigma \sigma$. The two directions s s and $\sigma \sigma$ clearly form, the one with the other, an angle of $180^0-2 \alpha$.

In order that the light may, alternately, be let in through the one or other plate, the entire refractometer is constructed so as to turn on its base, by which the observer is enabled to find the angle 180^{9} — 2α or 2α in a very short time without having to change the liquids.

In order to avoid the one reading on the divided circle, the refractometer is provided with a screw on which the telescope rests when the reading is of the right value for the leaving position. Thus, at the first adjustment, the telescope stands in a fixed position, so that the angle which corresponds to the refractory index is formed between the telescope and the partition in such a way that, by turning the prism, this adjusts itself to the fixed telescope.

If, according to Professor Hiortdahl's proposal, one selects 180° as the permanent reading value in the position of egress, then, on making the second adjustment, after turning the telescope from $180^{\circ}-2\alpha$, one will be able, clearly, to read off at over the angle 2α .

It is unnecessary to give a detailed description of the refractometer, as subsequent experience has shewn that the instrument employed in this instance, which was not, however, designed for sea water, can be reconstructed for the purpose, in such a manner that it gains, considerably, in utility, without losing anything in accuracy.

In order to carry out experiments, Dr. Hjort furnished me with 3 samples of sea water taken from the open sea. One of these (No. 5), in order to obtain water of the greatest known salinity, was left to evaporate in an open saucer for the night. The specific gravity of these 3 samples was obtained by means of Sprengel's Pyknometer, and under the observation of all possible precautionary measures. The amount of salt contained in the samples was computed from their specific weight, according to the following formula:

$$p = 1315 \left(S \frac{17^0.5}{17^0.5} - 1 \right)^*$$

From these 3 samples, 8 other salt water samples were formed by blending them with fixed quantities of distilled water, and their strength was thus very accurately known. Altogether there were, therefore, 11 salt water samples, from which the comparative observations in Table I, were prepared. Should the observations, subsequently, in practice, give good results, they

^{*)} See, Nyt Magazin for Naturvidenskaberne 34, pp. 235-236. The salinity is here given in thousandths.

must, clearly, be carried out in the temperature of their surroundings, that is to say, with varying temperatures from one time to the other. My observations had, therefore, to include, not only the measurement of strong and weak refractory powers of salt water, but also the measurement of the same sea water's refractory powers at different temperatures, in order to obtain the means for calculating the necessary corrections for temperature. Observations, which included salinity varying from 10.14 to $36.02^{0/00}$, and temperatures varying from $+ 5^{0.9}$ to $21^{0.5}$, will be found compared in Table I (all that is necessary for the explanation of the two last columns in that table will be given below). All measurements are given in degrees Celsius (centigrade), and the angles of deflection have been obtained under sodium light.

Table I.

Angle of Deflection	Temperature	Salinity (calculated)	Difference
	Sea Water	l, p = 33.86,	
15^{0} 7'.3	7.1	33.84	0.02
5'.9	7.7	.83	03
54.3	8.1	.85	01
4'.6	8.6	86	00
6'.4	7.6	86	00
4'.7	8.2	82	04
3'.9	8.9	85	01
04.3	10.9	84	02
0'.6	10.9	87	-0.01
0'.2	11.0	85	01
$14^{0}54'.6$	14.0	77	09
55'.4	14.1	84	02
554.0	14.3	83	03
54'.7	14.8	86	00
514.3	17.4	85	01
51'.2	17.6	87	01
514.8	17.8	92	- 06
51'.1	17:7	86	00
51'.2	17.6	87	01
51,3	18.1	91	05
50′.9	18.3	89	03
504.7	18.7	91	05
47′ 3	21.1	82	04
46′8	21.1	79	07
47'.1	21.3	. 82	04
46'.7	21.5	81	05

Angle of Deflection	Temperature	Salinity (calculated)	Difference		
	Sea Water	II, $p = 34.64$.			
$15^{0}20^{\circ}.3$	5.9	34.62	0.02		
1848	6.5	61	03		
184.7	7.0	68	- 04		
17'.9	7.4	68	- 04		
16'.4	7.8	63	01		
12'.6	9.9	65	- 01		
12'.6	10.0	66	- 02		
12'.4	10.2	66	- 02		
11'.9	10.4	65	- 01		
54.8	14.8	70	- 06		
4.8	15.4	69	- 05		
4'.7	160	74	- 10		
4'.2	16.5	75	- 11		
2'.0	17.2	64	00 .		
1'9	17.5	66	- 02		
14.8	17.7	67	- 03		
1'.7	17.9	68	- 04		
1′.0	17.7	61	03		
0'.6	17.9	60	04		
0′.8	18.1	63	01		
0'.8	18.3	65	- 01		
$14^{0}57'.1$	20.6	54	10		
57',8	20.4	57	07		
584.3	20.3	60	04		
584,9	20.2	64	04		
	Sea Water I	II, $p = 31.96$.			
140 26'.0	17.4	31.96	0.00		
25'.8	17.6	97	- 01		
25:.8	17.8	97	- 01		
25′.6	17.9	96	00		
20.0			00		
	Sea Water I				
$14^{0}40'.7$	17.2	33.04	- 0.04		
40'.1	17.4	01	- 01		
394.9	17.6	01	- 01		
39′.6	17.9	01	- 01		
	Sea Water	V, p = 36.02.			
$15^{o}19^{\circ}.6$	17.0	35.98	0.04		
19'.6	17.4	36.02	00		
19'0	17.8	00	02		
19′.1	18.0	03	- 01		

Angle of Deflection	Temperature	Salinity (calculated)	Difference		
	Sea Water \	VI, p = 29.97.			
13º 58'.6	17.2	29.95	0.02		
58'.6	17.4	97	00		
58'.6	17.5	98	01		
		'			
	Sea Water V	711, $p = 25.02$.			
12046'.0	17.0	25.01	0.01		
45'.8	17.3	01	01		
45′.9	17.5	03	— 01		
	Sea Water V	7111, $p = 20.02$			
$11^{0}24'.4$	17.2	19.96	0.06		
24'.3	17.6	96	06		
	Sea Water	1X, p = 10.14.			
$8^{0}7'.5$	17.4	10.16	- 0.02		
7'.4	17.7	15	01		

As is known, the refractory indices of the sea water concerned, are calculated according to the formula

$$n-I = \frac{\sin^2 \alpha}{n_0^2 (n+I)}$$

in which α is the measured angle of deflection at the change of light from water to sea water in Hallwach's differential prism, and n the relative refractory index in respect to pure water, and n_0 the pure water's refractory index in respect to air. The given observations allow of calculating how the [found angles of deflection (refractory indices) are dependent on the strength of the salt in the sea water concerned, and on the temperature under which the measurement was made. As the calculation does not present any special difficulties, I find it unnecessary to repeat, here, the steps in it, and confine myself, without bringing forward any further grounds, to repeating the formulæ which, conformably, express the stated subordinate conditions.

Brigg. log.
$$\frac{n \cdot \frac{57 \cdot 5}{17 \cdot 5}}{p} = 6 \cdot 14340 - 0 \cdot 00006 p$$
$$n \frac{t}{t} - n \frac{17 \cdot 5}{17 \cdot 5} = 0 \cdot 216 \left(s \frac{t}{t} - s \frac{17 \cdot 5}{17 \cdot 5} \right)$$

In this $n \frac{t}{t}$ and $s \frac{t}{t}$, respectively, denote the relative refractory indices, and relative specific weight of salt water at $t^0 c$, in relation to pure water of the same temperature, and p the strength of the salt in thousandths. In calculating $s \frac{t}{t}$, Ekman's Tables, concerning the expansion of sea water (Kgl. Svenska Vetenskapsakademiens Handlingar 1870-1), together with Rosettis' Tables concerning pure water (Landolt Börstnein, 1st drift. 34), have been employed.

The above mentioned formulæ, are, naturally, not suitable in practice when results have to be quickly stated, and that with the least possible trouble. I have therefore framed Tables II and III, of which the former gives the salinity in thousand parts, directly expressed by the angle of deflection at 17°.5 celsius, and the other, the correction, in thousandths of salinity, which is to be used for deviations from the normal temperature (17.°5 Cels.). The use of the Tables is very simple, and needs no explanation.

Table II.

The connection between Double Angles of Deflection by means of the Sodium Light in Differential Prisms, according to Hallwachs, and the Salinity of Sea Water at 17 0.5 Celsius.

Angle of Deflection 2 α	00/		00/		00/(00/
Angle of effection 2	Salinity ⁰ /00	ä	Salinity ⁰ /00	ಶ	Salinity ⁰ /00	ಜ	Salinity 0/00
ngle ecti	mit	57	init	62	init	62	iii t
A)	Sall		Sal		Sal		Sal
	1						
$15^{0} 50'$	38.45	150 10'	35.29	14º 30'	32.27	130 50'	29.38
49	37	9	21	29	20	49	30
48	29	8	14	28	12	48	28
47	21	7	3 5. 06	27	32.05	47	16
46	13	6	34.98	26	31.97	46	09
45	38 05	5	91	25	90	45	29.02
44	37.96	4	83	24	83	44	28.95
43	88	3	75	23.	75	43	88
42	80	2	67	22	68	42	81
41	72	1	60	21	60	41	74
$15^{0} 40'$	64	150 0'	52	140 20'	53	130 40'	28.67
39	. 56	140 59'	44	19	46	180 204	27.30
38	48	58	37	18	39	130 0'	25.95
37	40	57	29	17	31	120 40'	24.64
36	32	56	22	16	24	120 20'	23.37
35	25	55	14	15	17	120 0'	22.12
34	17	54	34.07	14	10	110 40'	20.91
33	09	53	33.99	13	31.02	110 20'	19.73
32	37.01	52	92	12	30.95	110 0'	18.59
31	36.98	51	84	11	88	100 40'	17.49
15 ⁰ 30'	85	140 50'	77	140 10'	81	100 20'	16.42
29	77	49	69	9	73	100 0'	15.37
28	69	48	61	8	66	90 40'	14.37
27	61	47	54	7	59	90 20'	13.39
26	53	46	46	6	52	8 ₀ 0,	12.46
25	46	45	39	5	44	80 40'	11.55
24	38	44	31	4	87	80 20'	10.68
23	30	43	24	3	30	80 0'	9.84
22	22	42	16	2	23	j	
21	14	41	09	1	15		
$15^{0} \ 20'$	36.06	140 40'	33.01	140 0'	08		
19	35.98	39	32.94	130 59'	30.01	1	
18	91	38	86	58	29.94		
17	83	37	79	57	87		
16	75	36	71	56	80		
15	68	35	64	55	73		
14	60	34	57	54	66		
13	52	33	49	53	59		
12	44	32	42	52	52		
11	37	31	34	51	45		

Table III.

Corrections in Thousandths of the Salinity, for Deviation from the Normal Temperature.

Approximate Salinity 0/00	36.0	35 . 5	35.0	34.5	34.0	33.5	33,0	32.5	32.0	31.5	31.0	30.5	30.0
					C	orrect	ions	0/00					
9	}			ŀ							}		
Observed Temperature													
$6^{0} 0$	-1.44	-1.43	-1.41	1.39	1.38	— 1 .36	-1.35	-1.33	1.31	— 1.30	-1.28	-1.27	-1.25
2	-1.41			1	- 1 35	1 33	1.32	-1.30	-128	-1.27	- 1.25	— 1.24	-1.22
4		1.37	į.	-1.33	- 1.32	1.30	1.29	1.27	1.25	- 1.24	-1.22	-1.21	- 1.20
6		1.33		- 1.30	1.29	1.27	1.20	1.24	1.25	1.10	-1.20 -1.17	- 1.18	1.17
8 70 0	- 1	-1.30 -1.27	1	1.27	1.20	1.24	1.25	1.21 1.19	1.20	- 1.13 - 1.16	- 1.14	1.13	-1.14
$\frac{70.0}{2}$	1,20 1.95	1.21	1.20	- 1.24 - 1.91	-1.20 -1.20	1 18	-1.20	1.16	1.14	1.13	1.11	-1.10	1.09
$\frac{2}{4}$	1.22	1.21	1.19	- 1.18	-1.17	-1.15	- 1.14	-1.13	-1.11	-1.10	- 1.08	-1.07	1.06
6	-1.18	-1.17	- 1.16	-1.15	-1.14	-112	-1.11	- 1.10	1.09	 1.08	-1.06	-1.05	-1.03
8	-1.15	-1.14	1.13	-1.12	— 1.11	1.09	-1.08	-1.07	1.06	— 1.05	-1.03	-1.02	-1.01
$8^{0}.0$	-1.12	1 11	- 1.10	1.09	- 1.08	-1.06	-1.05	-1.04	1.03	 1.02	1.00	-0.99	0.98
2	- 1.09										- 0.97		
4	1.06	— 1.05	— 1.04	- 1.03	-102	-1.01	1.00	0.99	0.98	0.97	0.95	-0.94	
6	ì	- 1.03	-102	- 1.01	- 1.00	0 98	-0.97	0.96	0.95	-0.94	- 0.92	0.91	0.90
8	1.01	1.00	0.99	- 0.98	- 0.97	0.9a	0.94	0.95 0.01	0.92	0.81	-0.90 -0.87	0.89 0.86	- 0.00 - 0.85
$\frac{90.0}{2}$	-0.98	0.94	- 0.99	0.95	0 91	0.95	-0.92	0.91	- 0.90 - 0.87	0.69 0.86	- 0.84	-0.88	-0.82
4											-0.82		
6	-0.90										- 0.79		
8											-0.77		
$10^{0}.0$	0 84	0.83	-0.82	- 0.81	-0.80	-0.79	0 78	- 0.77	0.76	-0.75	-0.74	0.73	-0.72
2	0.81	-0.81	0.80	- 0.79	- 078	- 0.77	-0.76	0.75	- 0.74	-0.73	0.72	→ 0.71	0.70
4											0.69		
6											- 0.67		
8											-0.65		
1100											-0.63 -0.60		
$\frac{2}{4}$	- 0.68										-0.58		
6											0.56		
8	-0.61										0.53		
$12^{0.0}$	-0.58										- 0.51		
2											0.49		
4	0.54				i .						-0.47		ž.
6											0.45		
8	- 0.49	0 49	- 0 48	— 0 47	- 0.47	— 0.46	0.46	— 0 . 45	- 0.44	0.44	- 0.43	- 0.43	- 0,42

						1	1		1	1	1	ı	
Observed Temperature		Î											
Observed emperatu			1	1							ŀ		
pse													
0 Fer		1											
130.0	-0.47	-0.47	- 0.46	- 0.45	-0.45	0.44	- 0.44	- 0.43	-0.42	-0.42	-0.41	-0.41	-0.40
2	0 45	-0.45	-0.44	0.43	-0.43	0.42 -	-0.41	- 0.41	- 0.40	-0.39	- 0.38	0.38	 0 37
4			0.42		- 0.41	0.40	0.39	— 0.39 ·	0.38	0.37	0.36	-0.36	-0.35
6	-0.40	- 0.40	0.39	- 0.38	-0.38	-0.37	-0.37	- 0.36	- 0.35	- 0.35		-0.34	
8	0.38	0.38	— 0.37	- 0.36	- 0.36	- 0.35	- 0.35	- 0.34	-0.33	-0.33	-0.32	-0.32	- 0.31
$14^{0}.0$	0.36	0.36	 0.35	- 0.34 -	- 0.34			- 0.32				-0.30	-0.29
2	0.34	— 0.34	- 0 33	- 0.32	0.32	0.31		0.30				-0.28	-0.27
4	- 0.32	- 0.32	- 0 31	- 0.30	-0.30	-0.29	-0.29	-0.28	- 0.27	-0.27			-0.26
6	-0.29	-0.29	0.29	- 1		1	-0.27	-0.26	- 0.26	- 0.26		-0.25	- 0.24
8	- 0 27		0.27			 0.25 -		-0.24	-0.24	1	-0.23	-0.23	-0.22
$15^{0}.0$	0.25		0.25			1	- 0.23	-0.23		-0.22		-0.21	- 0.21
2	-0.23		- 0.22						0.20			-0.19	- 0,19
4	0.21		0.20					-0.19	0.18	- 0.18	-0.17	-0.17	
6	-0.18	0.18	- 0.18	-0.18 -	-0.18	-0.17	0.17		- 0 17				
8	-0.16	-0.16	- 0.16	- 0.16	- 0.16	— 0.15 -	-0.15	- 0.15	-0.15	0.15	- 014	- 0.14	
$16^{0}.0$			- 0 14										0.12
2			- 0.12								0.11		- 0.11
4	- 0.10	-0.10	- 0.10	-0.10	-0.10	-0.10	-0.10		0.10				
6			0.09										
8			- 0.07										
$17^{0}.0$	-0.05	1	- 0.05		1							0.05	0.03 0.03
2	0.03						_ :	- 0.03	_ :				- 0.03 - 0.01
4	- 0.01	-0.01		0.011		- 0.01		- 0.01	0.01		0.01 0.00	0.00	0.00
6 -	0 01	0.01	0.01	0.01	0 01	0 01	0.01	0.01	0.00	0.00	0.00	0.00	0.01
8	0 02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.03	0.03	0.03
$18^{0.0}$	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.03	0.03	0.04	0.04	0.04
2	0.06	0.06	0.06	0.06	0.06	0.05	0.05		0.04	0.06	0.05	0.05	0.05
4	0.07	0.07	0.07	0.07	0.07	0.07	$0.07 \\ 0.08$	$0.07 \\ 0.08$	0.06	0.07	0.07	0.07	0.06
6	0.09	0.09	0.09	0.09	0.09	0.08	0.09	0.09	0.08	0.08	0.08	0.08	0.07
100.0	0 11		0.10	$0.10 \\ 0.12$	$0.10 \\ 0.12$	$0.10 \\ 0.11$	0.03	0.03	0.10	0.10	0.09	0.09	0.09
$19^{0}.0$	0.13	0.13	$0.12 \\ 0.14$	0.12	0.12	0.11	0.12	$0.11 \\ 0.12$	0.11	0.11	0.10	0.10	0.10
2	0 14	0.14	0.14	0.15	0.15	0.14	0.12	0.13	0.12	0.12	0.11	0.11	0.11
4	0.16	0.16	0.17	0.16	0.16	0:15	0.14	0.14	0.13	0.13	0.13	0.13	0.12
6 8	0.18	$0.18 \\ 0.19$	0.18	0.18	0.18	0 17	0.16	0.16	0.15	0 15	0.14	0.14	0.13
200.0	0.19	0.15	0.10	0.19	0.19	0.18	0.17	0.17	0.16	0.16	0.15	0.15	0.14
	0.21		0.20	0.21	0.20	0.19	0.18	0.18	0.17	0.17	0.16	0.16	0.15
$\frac{2}{4}$	0.23		0 23	0.22	0.20	0.21°	0.20	0.20	0.18	0.18	0.17		0.16
6	0 24			0.24	0.23	0.22	0.21	0.21	0.20	0.19	0.18	0.18	0.17
8	0.27			0.25	0.25	0.24	0.23	0.22	0.21	0.20	0.19	0.19	0.18
210.0	0.29			0.27	0.26	0.25	0.24	0.24	0.22	0.22	0.21	0.20	0.19
21.0	0.31			0 28	0.27	0.26	0.25	0.25	0.23	0.23	0.22	0.21	0.20
4	0.32			0 30	0.29	0.28	0.27	0.26	0.24	0.24	0.23	0.22	0.21
6	0.34			0.31	0.30	0.29	0.28	0.27	0.26	0.25	0.24	0.23	0.22
8	0.35		: !	0 33	0.32	0.31	0.30	0.29	0.27	0.26	0.25	0.24	0.23
220,0	0.37	0.36		0 34	0.33	0.32	0.31	0.30	0.28	0.27	0.26	0.25	0.24
	, 0.0.												

By the aid of Tables II and III, the third columns in Table I, have been calculated. The true strength of salt, as found by its specific gravity, or by blending it with pure water, is, in the said table, given above, at the head of the row of observations which refer to the salt water sample concerned. The Difference between the thus given true salinity, and the salinity calculated from the angles of deflection, are to be found in the 4th column of Table I.

As the observations are somewhat numerous (73), and that, in order to satisfy so many observations in respect to the above given equations, one can but command the use of 3 — three — constants, it is thereby evident that, the 4th column in Table I, is a very good test of what the method can do in the way of exactitude. Great errors, exceeding $0.07^{0}/00$, only occur 4 times, and in 59, out of 73 observations, the error is less than $0.05^{0}/00$. The average of error is $0.027^{0}/00$. It may be remarked that I have been less favourably situated than one working under natural conditions, as I was obliged to make the observations at the temperature necessary for the solution of my task, and not in that of the surroundings. From this, it may be regarded as proved that, the method allows of very considerable accuracy. The completion of an observation demands but a few moments.

Corresponding observations, and calculating tables can, naturally, be employed in determining the strength of other resolvents (for instance Titration solutions, and might be of importance in ordinary chemical laboratory practice.

Kristiania, February 1900.